Equation Sheet for EMch 112H
Mechanics of Motion

Miscellaneous

If $ax^2 + bx + c = 0$, then $x = \left(-b \pm \sqrt{b^2 - 4ac}\right) \Big/ 2a$.

Rectilinear (1-D) Motion

Position: $s(t)$; Velocity: $v = \dot{s} = ds/dt$; Acceleration: $a = \ddot{s} = dv/dt = d^2s/dt^2 = v dv/ds$. For *constant acceleration* a_c:

$$v^2 = v_0^2 + 2a_c(s - s_0) \qquad v = v_0 + a_c t \qquad s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

There are corresponding equations for constant angular acceleration α.

2D Motions—Cartesian Coordinates

Position: $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath}$; Velocity: $\vec{v} = d\vec{r}/dt = \dot{x}\,\hat{\imath} + \dot{y}\,\hat{\jmath}$; Acceleration: $\vec{a} = d\vec{v}/dt = d^2\vec{r}/dt^2 = \ddot{x}\,\hat{\imath} + \ddot{y}\,\hat{\jmath}$

2D Motions—Normal-Tangential (Path) Coordinates

$$\vec{v} = v\,\hat{e}_t = \rho\dot{\beta}\,\hat{e}_t \qquad \vec{a} = \dot{v}\,\hat{e}_t + (v^2/\rho)\,\hat{e}_n,$$

2D Motions—Polar Coordinates

$$\vec{r} = r\,\hat{e}_r \qquad \vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta \qquad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$$

Relative Motion

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad \vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Newton's Second Law

$$\vec{F} = m\vec{a}; \quad F_x = ma_x; \quad F_y = ma_y; \quad F_n = ma_n; \quad F_t = ma_t; \quad F_r = ma_r; \quad F_\theta = ma_\theta$$

Work-Energy Principle

$$U_{1\text{–}2} = \int_{\text{path}} \vec{F} \cdot d\vec{r} \qquad T = \tfrac{1}{2}mv^2 \qquad T_1 + U_{1\text{–}2} = T_2$$

$$V_g = mgh \qquad V_e = \tfrac{1}{2}k\delta^2 \qquad T_1 + V_{g1} + V_{e1} + U'_{1\text{–}2} = T_2 + V_{g2} + V_{e2}$$

Linear Impulse-Momentum Principle

$$\vec{G} = m\vec{v} \qquad \vec{F} = \dot{\vec{G}} \qquad m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2$$

Angular Impulse-Momentum Principle

$$\vec{H}_O = \vec{r}_{P/O} \times m\vec{v}_P \qquad \vec{M}_O = \dot{\vec{H}}_O \qquad \vec{H}_{O_1} + \int_{t_1}^{t_2} \vec{M}_O\,dt = \vec{H}_{O_2}$$

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Impact of Smooth Particles

$$m_1(v_1)_n + m_2(v_2)_n = m_1(v_1')_n + m_2(v_2')_n; \quad (v_1)_t = (v_1')_t; \quad (v_2)_t = (v_2')_t; \quad e = \frac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n}$$

Systems of Particles

$$\sum \vec{F} = m\vec{a}_G \qquad T_1 + V_1 + U'_{1\text{–}2} = T_2 + V_2$$

$$T = \tfrac{1}{2}mv_G^2 + \sum \tfrac{1}{2}m_i|\dot{\vec{\rho}}|^2 \qquad \vec{G} = m\vec{v}_G$$

$$\sum \vec{M}_P = \dot{\vec{H}}_G + \vec{r}_{G/P} \times m\vec{a}_G \quad \text{(M\&K)} \qquad \sum \vec{M}_P = (\dot{\vec{H}}_P)_{\text{rel}} + \vec{r}_{G/P} \times m\vec{a}_P \quad \text{(M\&K)}$$

$$\sum \vec{M}_P = \dot{\vec{H}}_P + m\dot{\vec{r}}_P \times \dot{\vec{r}}_G \quad \text{(GLG)} \qquad \sum \vec{M}_P = \dot{\vec{h}}_P + m\vec{\rho}_G \times \ddot{\vec{r}}_P \quad \text{(GLG)}$$

Under the appropriate conditions, the GLG moment-angular momentum relations simplify to: $\sum \vec{M}_P = \dot{\vec{H}}_P$ or $\sum \vec{M}_P = \dot{\vec{h}}_P$

Rigid Body Kinematics

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} \qquad \vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

Moments of Inertia

$$(I_G)_{\text{disk}} = \tfrac{1}{2}mr^2 \qquad (I_G)_{\text{rod}} = \tfrac{1}{12}ml^2 \qquad (I_G)_{\text{plate}} = \tfrac{1}{12}m(a^2 + b^2) \qquad (I_G)_{\text{sphere}} = \tfrac{2}{5}mr^2$$

Parallel Axis Theorem: $I_A = I_G + md^2$; Radius of Gyration: $I_A = mk_A^2$

Angular Momentum and Equations of Motion for a Rigid Body

Angular Momentum

$$\vec{H}_G = I_G\vec{\omega} \qquad \vec{H}_O = I_G\vec{\omega} + \vec{r}_{G/O} \times m\vec{v}_G$$

Equations of Motion

The *translational equations* are given by

$$\sum \vec{F} = m\vec{a}_G$$

The *rotational equation*. For the mass center G: $\sum M_G = I_G\alpha$ and for a fixed point O: $\sum M_O = I_O\alpha$. For an arbitrary point A:

$$\sum \vec{M}_P = I_G\vec{\alpha} + \vec{r}_{G/P} \times m\vec{a}_G \qquad \sum \vec{M}_P = I_P\vec{\alpha} + \vec{r}_{G/P} \times m\vec{a}_P \qquad \left(\vec{M}_A\right)_{\text{FBD}} = \left(\vec{M}_A\right)_{\text{KD}}$$

Work-Energy for a Rigid Body

The work-energy principle is the same as that for particles. The kinetic energy of a rigid body is:

$$T = \tfrac{1}{2}I_O\omega^2 \qquad T = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}I_G\omega^2$$