Torsion: Power Transmission and Stress Concentrations Design Supplement

This overview of the theory for design of power transmission shafts is meant to supplement that given in your textbook. It covers power transmission, stress concentrations and design of round shafts.

Power Equations

We can model shafts rotating at constant angular velocity using statics because we know from dynamics that when angular acceleration equals zero, $\sum T = 0$. Knowing the speed of rotation, we can convert between torque T and power P using the equations given in the text, namely

$$T = \frac{P}{\omega}, \qquad \omega = 2\pi f, \qquad \therefore T = \frac{P}{2\pi f} \tag{1}$$

where power is measured as torque per unit time and ω and f are the angular velocities usually given in radians per second and revolutions per second, respectively. We deliberately cast these equations to solve for torque because this is useful in stress analysis. Furthermore, f is the cyclic frequency which given per second is known as Hertz (Hz) or cycles per second and given per minute is known as RPM or revolutions per minute. The former is commonly associated with Watts or N·m/s and the latter with horsepower (hp), a U.S. Customary Unit, in which case,

$$T(\mathrm{ft \cdot lb}) = \left(\frac{550\,\mathrm{ft \cdot lb}}{\mathrm{hp \cdot s}}\right) \times \left(\frac{60\mathrm{s}}{\mathrm{min}}\right) \times \left(\frac{HP}{2\pi RPM}\right) \approx 5250\left(\frac{HP}{RPM}\right) \tag{2}$$

Here torque is specifically calculated in ft·lb units. (Note: To distinguish between like variables and units, we employ upper case italic for variables, lower case standard for units.)

Stress Concentration Equations

Stress concentrations in torsion arise when the geometry is interrupted. For example, Hibbeler (1997) provides stress concentration factors for shoulder fillets in shafts. Other concentrators are caused by keyways (Boresi and Sidebottom, 1985; Machinery's Handbook, 1996), grooves and drill-through holes (Roark and Young, 1975; Machinery's Handbook, 1996). When applying formulas from these sources, it is important to note the definition of the nominal or reference stress and the stress concentration factor itself.

Design Applications

In design applications we typically need to specify shaft dimensions such as a diameter and fillets. The design is determined with respect to allowable stress (strength) and sometimes twist.

For pure torque loading, it is convenient to rewrite the stress-from-torque expression as

$$\tau = \frac{Tr}{J} = \frac{Tr}{\pi r^4/2} = \frac{2T}{\pi r^3} = \frac{16T}{\pi d^3} \qquad \Rightarrow \qquad d = \left(\frac{16T}{\pi\tau}\right)^{1/3} \tag{3}$$

where r and d denote the radius and diameter of the shaft, respectively. Moreover in the absence of other factors, we would replace the shear stress by an allowable value found using a factor of safety together with material test data. (See the problems below.)

If a complication such as a stress concentrator exists, then a design problem may become iterative if initially we do not have the shaft geometry from which to determine the stress concentration factor. Then the design algorithm is: 1) solve the problem for dimensions, 2) use the stress concentration factor to check if allowables are violated, and 3) repeat until optimal dimensions which satisfy all requirements are found.

In the case of power take-off applications which typically involve belts or chains wrapped around sheaves or gears, bending as well as torsion occurs. This is treated by Hibbeler (1997) in a section devoted to shaft design.

Interestingly, Machinery's Handbook (1996, p. 280) recommends for allowable stresses 60% of σ_Y for axial and 30% of σ_Y for shear where σ_Y is the yield stress in tension. This amounts to factors of safety *FS* (assume that yield in shear is 60% tensile yield) of

$$FS_{\text{axial}} = \sigma_Y / 0.6\sigma_Y = 1.7 \qquad FS_{\text{shear}} = 0.6\sigma_Y / 0.3\sigma_Y = 2.0 \tag{4}$$

which are rather high values indicating shaft design is viewed very conservatively. An official standard is not cited.

Examples

TD1. A solid SAE 1018 hot-rolled steel shaft is required to transmit 2 hp at 1725 rpm. Specify the minimum diameter shaft to the nearest 1/32 in if τ_{allow} = 18 ksi.
Solution:

1. $\omega = 1725\text{rpm}\left(\frac{2\pi\text{rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\text{s}}\right) = 180.6$ rad/s

2. $P = 2\text{hp}\left(550\frac{\text{ft}\cdot\text{lb}}{\text{s}\cdot\text{hp}}\right)12\frac{\text{in}}{\text{ft}} = 13200$ in lb/s

3. $T = \frac{P}{\omega} = \frac{13200}{180.6} = 73.09$ in lb, or from (2) $T = \frac{5250(2)12}{1725} = 73.04$ in lb. O.K.

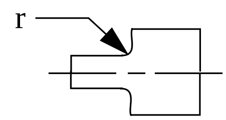
4. $d = \left(\frac{16T}{\pi\tau}\right)^{1/3} = 0.2745$ in. Hence choose d = 9/32 in. ♦

5. Check 1: $\tau = \frac{Tr}{J} = \frac{2T}{\pi r^3} = \frac{2(73.04)}{\pi(9/64)^3} = 16,730 \text{ psi} < 18,000 \text{ psi}$ Hence O.K.

6. Check 2: $\tau = \frac{Tr}{J} = \frac{2T}{\pi r^3} = \frac{2(73.04)}{\pi(8/64)^3} = 23,800 \text{ psi}$ 1/4 in dia. shaft is too small.

7. Decision: Select SAE 1018 hot-rolled steel bar, 9/32 in dia. From step 5, the design is within 93% of the allowable stress.

TD2. The shaft in problem TD1 is changed to include a shoulder which abuts against a bearing. The smaller diameter remains 9/32 in; the larger is required to be 5/8 in. Determine the minimum fillet radius in the shoulder. The power transmitted is the same and axial loading can be ignored.



Solution:

1. Let d = 9/32 in, D = 5/8 in. Hence D/d = 1.11
2. $K = \frac{\tau_{max}}{Td/(2J)} = \frac{18,000}{16,730} = 1.08 \approx 1.1$ Note: use d not D to calculate the nominal stress.
3. From a reference having stress concentration factors for a shaft with a shoulder, we can obtain the minimum fillet radius. For instance, using Hibbeler (1997, Fig. 5-36), we are off the chart, but extrapolation yields $r/d > 0.33$ or r > 3/32 in. Clearly the fillet will be much smoother than indicated by the illustration above. Furthermore, proper mounting of the bearing may also influence the choice of fillet radius. Such indefiniteness is common in practice. Solutions require engineering experience. (Note: Hibbeler's definition and chart axis reference the smaller of the two shaft diameters.)

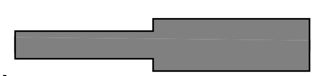
Problems

TD3. A solid, 2014T6 aluminum shaft is required to transmit 2 hp generated by a single-phase electric motor turning at 1725 rpm. If there are no gear reductions involved, specify the minimum diameter shaft to the nearest 1/32 in if the factor of safety is 1.2. (ans: dia. = 9/32 in)

TD4. a) Derive a formula analogous to (3) for the outer diameter of a hollow shaft in terms of the inner diameter and other variables.

b) Specify the inner diameter to the nearest 1/32 in of a hollow shaft with 5/16 in outer diameter to satisfy the requirements of problem TD3.

TD5. Specify the minimum diameters of the solid 2014T6 aluminum step shaft rotating at 2.5 Hz under 28 kW input at C and 15 and 13 kW taken off at B and A, respectively. Factor of safety will not be accounted for in calculating these minima.



References

Boresi, Arthur P. and O.M. Sidebottom (1985) Advanced Mechanics of Materials, John Wiley & Sons, New York.

Machinery's Handbook (1996) 25th ed., Industrial Press Inc., New York.

Hibbeler, R.C. (1997) Mechanics of Materials, 3rd ed., Prentice Hall, Englewood Cliffs.

Roark, Raymond J. and W.C. Young (1975) Formulas for Stress and Strain, 5th ed., McGraw-Hill Book Co., New York.