Introduction to Continuum Mechanics

Fall 2001

Instructor: Dr. Francesco Costanzo–409D Earth & Engineering Sciences Bldg.
Phone: (814) 863-2030
E-mail: costanzo@engr.psu.edu
Office Hours: MWF at 10:00 A.M.–11:00 A.M. or by appointment

Required Texts: P. Chadwick, *Continuum Mechanics: Concise Theory and Problems*, Dover Publishing, 1999 (first edition: Wiley, 1976).

A. J. M. Spencer, *Continuum Mechanics*, Longman, 1980.

Additional References

1. P. Haupt, *Continuum Mechanics and Theory of Materials*, Springer-Verlag, 2000.
2. R. M. Bowen, *Introduction to Continuum Mechanics for Engineers*, Plenum Press, 1989.
3. M. E. Gurtin, *An Introduction to Continuum Mechanics*, Academic Press, 1989.

Course Objective

This course provides the fundamental concepts and methods used in the mathematical modeling of solids and fluids. Many material systems, regardless of their nanostructure, can be considered **continuous** from a macroscopic viewpoint. This apparently simple abstraction is, in reality, the cornerstone of most of the mathematical models of materials used in engineering.

The course will cover the fundamental aspects of continuum theories: (*i*) the description of deformation (kinematics), (*ii*) the basic conservation laws, and (*iii*) the description of the constitutive behavior of both fluids and solids. Both nonlinear and linear kinematics is treated as well as index and direct notations. Conservation principles will be derived according to both the Lagrangian and Eulerian frameworks.

Special attention will be paid to the relation between constitutive equations and the form of resulting boundary value problems. Applications will include both nonlinear and linear elasticity as well as compressible and incompressible fluids. In the case of elastic solids, the analysis of material symmetry will be covered so as to be applicable to anisotropic materials (composites).

New Teaching Approach

A new approach to the teaching of Continuum Mechanics will be used. In particular, the course presentation will heavily rely on graphic animations to illustrate all of the various aspects of the course. These animations have been created using the symbolic and numerical analysis package *Mathematica*. The software with which the animations are created will be made available to all of the students in the course to allow them to design their own interactive virtual experiments. This will allow students to interactively create and animate deformations of a material sample as well as visualize the deformation of a material as it derives from an assigned constitutive theory. The use of *Mathematica* will be encouraged in the solution of homework problems. If necessary, a few tutorial sessions on the use of *Mathematica* will be organized.

Grading Scheme and Exams

The overall grade in the course will consists of the average of the grades gained in the following three activities:

1. Homework (worth 1/3 of the overall grade);
2. One midterm take-home exam (worth 1/3 of the overall grade); and
3. A take-home final exam (worth 1/3 of the overall grade).

Course Outline

1. **Mathematical Preliminaries**
   - Index and Direct Notations
   - Tensor Algebra and Calculus
   - Divergence and Stokes' Theorems
2. **Kinematics**
   - Deformation and Deformation Rates
   - Transformation of Linear Surface and Volume Elements
   - Finite and Infinitesimal Strain Tensors
3. **Conservation Laws and Thermodynamics**
   - Balance of Mass
   - Balance of Linear and Angular Momentum
   - Balance of Energy
   - Theorem of Virtual Work
   - First Law of Thermodynamics (Balance of Energy)
   - Second Law of Thermodynamics (Entropy Production Inequality)
4. **Classical Constitutive Equations**
   - Isothermal Nonlinear Elastic Solid
   - Isothermal Linearized Elasticity
   - Newtonian Fluid
   - The Viscous Heat Conducting Fluid
5. **General Aspect of Constitutive Theory**
   - Invariance Requirements
   - Thermodynamics Constraints
   - Material Symmetry
   - Special Kinematic Assumptions
6. **Kinematics and Balance Laws for Discontinuous Processes**
   - Moving Interfaces
   - Jump Conditions of the Basic Balance Laws