Handedness reversal of circular Bragg phenomenon due to negative real permittivity and permeability

Akhlesh Lakhtakia

CATMAS, Department of Engineering Science and Mechanics, Pennsylvania State University, University Park, PA 16802-6812

AXL4@psu.edu

Abstract: When the real parts of the permittivity and the permeability dyadics of a structurally chiral, magnetic-dielectric material are reversed in sign, the circular Bragg phenomenon displayed by the material is proved here to suffer a change which indicates that the structural handedness has been, in effect, reversed. Additionally, reflection and transmission coefficients suffer phase reversal.

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References and links

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1. Introduction

Constructive and destructive interference of the consequences of electromagnetic processes occurring in different parts of a periodically nonhomogeneous material is responsible for the Bragg phenomenon. From a mathematical viewpoint, the interaction of the spatial harmonics of the electromagnetic field and the spatial harmonics of the permittivity (or any other relevant constitutive property) can become resonant [1], thereby leading to virtually total reflection in certain, fairly narrow, wavelength-regimes.

The Bragg phenomenon is widely exploited for filtering by cascades of alternating homogeneous layers of two materials of different permittivities [2]. Provided that both types of layers are isotropic, and light is normally incident, the Bragg phenomenon is insensitive to the polarization state of the incident light. That insensitivity generally vanishes with the introduction of anisotropy (although special cases of virtually no sensitivity can still be conceived of [3]).

A particularly interesting feature — discrimination between incident left and right circularly polarized (LCP and RCP) plane waves — emerges when the Bragg phenomenon is displayed by chiral liquid crystals [4] and chiral sculptured thin films [5, 6]. These anisotropic materials are structurally chiral (i.e., handed) and unidirectionally nonhomogeneous. When the wave vector of the incident plane wave is parallel to the direction of nonhomogeneity, extremely high reflection is observed if the handedness of the incident light coincides with the structural handedness of the material; and extremely low reflection is observed if the two handednesses do not match. This circular-polarization-sensitivity gives rise to the term circular Bragg phenomenon, and underlies the many optical applications of these materials [4, 7]. In the time domain, the circular-polarization-sensitive high reflection manifests itself as a pulse-bleeding phenomenon [8] — which becomes clear from viewing movies of a pulse moving across the planar interface of a structurally chiral material and free space [9]. The circular Bragg phenomenon (accompanied by less significant analogs in one lower and many higher frequency regimes) also occurs for incidence in other directions [10, 11].

Structurally chiral materials can have magnetic properties in addition to dielectric [12]. For normal incidence on slabs made of the simplest of these type of materials with both magnetic and dielectric properties — called ferrocholesteric materials [13] — the circular Bragg phenomenon was recently shown [14] to reverse its circular-polarization-sensitivity when the real parts of its permeability and permittivity dyadics are changed from positive to negative. That observation is extended here for oblique incidence on chiral ferrosmectic slabs [15], a far more general scenario; and a mathematical proof is provided. This work is obviously inspired by the recent spate of papers published on the inappropriately designated left-handed materials which are macroscopically homo-
geneous and display negative phase velocities, but are not chiral [16, 17].

A note about notation: Vectors are in boldface, dyadics are double underlined, column vectors are underlined and enclosed in square brackets, while matrices are double underlined and also enclosed in square brackets. An \( \exp(-i\omega t) \) time dependence is implicit, with \( \omega \) as the angular frequency; \( \epsilon_0 \) and \( \mu_0 \) are the free-space permittivity and permeability, respectively; \( k_0 = \omega(\epsilon_0\mu_0)^{1/2} \) is the free-space wavenumber and \( \lambda_0 = 2\pi/k_0 \) is the free-space wavelength. A cartesian coordinate system is used, with \( \mathbf{u}_x, \mathbf{u}_y \) and \( \mathbf{u}_z \) as the cartesian unit vectors.

2. Theory

The nonhomogeneous permittivity and the permeability dyadics of a chiral ferrosmectic slab of thickness \( L \) are given as

\[
\epsilon(r) = \epsilon_0 \sum_{\text{sym}} \left[ \epsilon_a \mathbf{u}_z \mathbf{u}_z + \epsilon_b \mathbf{u}_x \mathbf{u}_x + \epsilon_c \mathbf{u}_y \mathbf{u}_y \right] \cdot \sum_{T} \left[ \mathbf{S}_{0}^T \cdot \mathbf{S}_{0} \right] \}
\]

\[
\mu(r) = \mu_0 \sum_{\text{sym}} \left[ \mu_a \mathbf{u}_z \mathbf{u}_z + \mu_b \mathbf{u}_x \mathbf{u}_x + \mu_c \mathbf{u}_y \mathbf{u}_y \right] \cdot \sum_{T} \left[ \mathbf{S}_{0}^T \cdot \mathbf{S}_{0} \right] \}
\]

where the superscript \( T \) denotes the transpose. The tilt dyadic \( \mathbf{S}_{0} = \mathbf{u}_y \mathbf{u}_y + (\mathbf{u}_x \mathbf{u}_x - \mathbf{u}_z \mathbf{u}_z) \cos \chi + (\mathbf{u}_z \mathbf{u}_z - \mathbf{u}_x \mathbf{u}_x) \sin \chi \) is a function of the angle \( \chi \in \lbrack 0, \pi/2 \rbrack \). The rotation dyadic \( \mathbf{S}_{0} = \mathbf{u}_y \mathbf{u}_y + (\mathbf{u}_x \mathbf{u}_x + \mathbf{u}_y \mathbf{u}_y) \cos \zeta + (\mathbf{u}_x \mathbf{u}_x - \mathbf{u}_y \mathbf{u}_y) \sin \zeta \), with \( \zeta = h\pi z/\Omega \), involves \( 2\Omega \) as the structural period along the \( z \) axis. The parameter \( h \) is allowed only two values: \(+1\) for structural right-handedness, and \(-1\) for structural left-handedness.

The complex-valued scalars \( \epsilon_{a,b,c} \) and \( \mu_{a,b,c} \) are functions of \( \omega \). For later use, \( \tilde{\epsilon}_d = \epsilon_a\mu_0/(\epsilon_a \cos^2 \chi + \epsilon_b \sin^2 \chi) \) and \( \tilde{\mu}_d = \mu_a\mu_0/(\mu_a \cos^2 \chi + \mu_b \sin^2 \chi) \) are defined.

Without loss of generality, the field phasors everywhere can be written as

\[
\begin{align*}
\mathbf{E}(r) &= \tilde{\mathbf{E}}(z) \exp \lbrack i\kappa(x \cos \psi + y \sin \psi) \rbrack \\
\mathbf{H}(r) &= \tilde{\mathbf{H}}(z) \exp \lbrack i\kappa(x \cos \psi + y \sin \psi) \rbrack
\end{align*}
\]

where \( \kappa \in \lbrack 0, \infty \rbrack \) and \( \psi \in \lbrack 0, 2\pi \rbrack \). The fields inside the chiral ferrosmectic slab must follow the \( 4 \times 4 \) matrix ordinary differential equation [12]

\[
\frac{d}{dz} \left[ \begin{array}{c} f(z) \\ \mathbf{P}(z) \end{array} \right] = i \left[ \begin{array}{c} f(z) \\ \mathbf{P}(z) \end{array} \right] , \quad 0 < z < L .
\]

In this equation, \( \left[ f(z) \right] = \left[ \tilde{E}_x(z), \tilde{E}_y(z), \tilde{H}_x(z), \tilde{H}_y(z) \right]^T \) is a column vector, while the \( 4 \times 4 \) matrix function \( \left[ \mathbf{P}(z) \right] \) is specified as follows:

\[
\left[ \mathbf{P}(z) \right] = \left[ \mathbf{P}_0(z) \right] + \frac{\kappa}{k_0} \left[ \mathbf{P}_0(z) \right] + \left( \frac{\kappa}{k_0} \right)^2 \left[ \mathbf{P}_0(z) \right] ,
\]

\[
\left[ \mathbf{P}_0(z) \right] = \frac{\omega}{2} \left[ \begin{array}{cccc}
0 & 0 & 0 & \mu_0 \frac{\tilde{\epsilon}_d + \tilde{\mu}_d}{2} \\
0 & 0 & -\mu_0 \frac{\tilde{\epsilon}_d - \tilde{\mu}_d}{2} & 0 \\
\epsilon_0 \frac{\tilde{\epsilon}_d - \tilde{\mu}_d}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\mu_0 \frac{\tilde{\epsilon}_d - \tilde{\mu}_d}{2} \\
-\epsilon_0 \frac{\tilde{\epsilon}_d + \tilde{\mu}_d}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\mu_0 \frac{\tilde{\epsilon}_d - \tilde{\mu}_d}{2} \\
\epsilon_0 \frac{\tilde{\epsilon}_d - \tilde{\mu}_d}{2} & 0 & 0 & 0 \\
-\epsilon_0 \frac{\tilde{\epsilon}_d + \tilde{\mu}_d}{2} & 0 & 0 & 0 \\
\end{array} \right] .
\]
can be stated in matrix form as

\[ P(z) = (k_0 \sin \chi \cos \chi) \times \]

\[
\begin{bmatrix}
\cos \zeta \cos \psi & 0 & 0 & 0 \\
0 & \sin \zeta \sin \psi & 0 & 0 \\
0 & 0 & \sin \zeta \sin \psi & 0 \\
0 & 0 & 0 & \cos \zeta \cos \psi \\
\end{bmatrix}
\]

\[ + \begin{bmatrix}
\sin \zeta \sin \psi & 0 & 0 & 0 \\
0 & \cos \zeta \cos \psi & 0 & 0 \\
0 & 0 & \cos \zeta \cos \psi & 0 \\
0 & 0 & 0 & \sin \zeta \sin \psi \\
\end{bmatrix} \]

\[ + \begin{bmatrix}
\cos \zeta \sin \psi & 0 & 0 & 0 \\
0 & \cos \zeta \sin \psi & 0 & 0 \\
0 & 0 & \cos \zeta \sin \psi & 0 \\
0 & 0 & 0 & \cos \zeta \sin \psi \\
\end{bmatrix}
\]

Equation (3) can be solved in a variety of ways, mostly numerical [11, 12, 18]. The result can be stated in matrix form as

\[ [f(L)] = [M] [f(0)] , \]

where \([M]\) is a 4×4 matrix.

Suppose the half-spaces \(z \leq 0\) and \(z \geq L\) are vacuous. Let the incident, reflected and transmitted plane waves be represented by

\[ e_{inc}(r) = \left( \frac{(is - p_+)}{\sqrt{2}} a_L - \frac{(is + p_+)}{\sqrt{2}} a_R \right) e^{ik_0 z \cos \theta} , \quad z \leq 0 , \]

\[ e_{ref}(r) = \left( \frac{(is - p_-)}{\sqrt{2}} r_L + \frac{(is + p_-)}{\sqrt{2}} r_R \right) e^{-ik_0 z \cos \theta} , \quad z \leq 0 , \]

\[ e_{tr}(r) = \left( \frac{(is + p_+)}{\sqrt{2}} t_L - \frac{(is - p_-)}{\sqrt{2}} t_R \right) e^{ik_0 (z-L) \cos \theta} , \quad z \geq L , \]

respectively, where \(\theta = \sin^{-1}(\kappa/k_0)\), while the unit vectors \(s = -u_x \sin \psi + u_y \cos \psi\) and \(p_{\pm} = \mp (u_x \cos \psi + u_y \sin \psi) \cos \theta + u_z \sin \theta\). The amplitudes of the LCP and the RCP components of the incident plane wave, denoted by \(a_L\) and \(a_R\), respectively, are assumed given. The four unknown amplitudes \(r_L, r_R, t_L\) and \(t_R\) of the circularly polarized components of the reflected and transmitted plane waves are determined by solving a boundary value problem obtained by enforcing the boundary conditions on the interfaces \(z = 0\) and \(z = L\) and using (9)-(11) in (8). The result is best put in terms of reflection coefficients \((r_{LL}, \ etc.)\) and transmission coefficients \((t_{LL}, \ etc.)\) appearing in the 2×2 matrices on the right sides of the following relations:

\[
\begin{bmatrix}
r_L \\
r_R
\end{bmatrix}
= \begin{bmatrix}
r_{LL} & r_{LR} \\
r_{RL} & r_{RR}
\end{bmatrix}
\begin{bmatrix}
a_L \\
a_R
\end{bmatrix}
, \quad
\begin{bmatrix}
t_L \\
t_R
\end{bmatrix}
= \begin{bmatrix}
t_{LL} & t_{LR} \\
t_{RL} & t_{RR}
\end{bmatrix}
\begin{bmatrix}
a_L \\
a_R
\end{bmatrix} .
\]
Consistently with the theme Negative Refraction and Metamaterials of this special issue, the intention here is to present a very significant observable consequence of the transformation

\[
\begin{align*}
\text{Re} \left[ \varepsilon(\mathbf{r}) \right] &\to - \text{Re} \left[ \varepsilon(\mathbf{r}) \right], \\
\text{Re} \left[ \mu(\mathbf{r}) \right] &\to - \text{Re} \left[ \mu(\mathbf{r}) \right], \\
0 &\leq z \leq L
\end{align*}
\]  
\tag{13}
\]

This consequence is evident in the plots of the reflectances \( R_{LL} = |r_{LL}|^2 \), etc. shown in Fig. 1. The wavelength-range for this figure was chosen to focus on the circular-polarization-sensitivity of the circular Bragg phenomenon, by an examination of three cases:

(i) For a structurally right-handed ferrosmectic slab \((h = +1)\) with all components of \(\text{Re} \left[ \varepsilon(\mathbf{r}) \right]\) and \(\text{Re} \left[ \mu(\mathbf{r}) \right]\) positive, a high-reflectance ridge is evident in the plots of \(R_{RR}\) accompanied by virtually null-valued \(R_{LL}\) and very small \(R_{RL} = R_{LR}\).

(ii) For a structurally left-handed ferrosmectic slab \((h = -1)\) with all components of \(\text{Re} \left[ \varepsilon(\mathbf{r}) \right]\) and \(\text{Re} \left[ \mu(\mathbf{r}) \right]\) positive, the circular-polarization-sensitivity is reversed. A high-reflectance ridge is evident in the plot of \(R_{LL}\) accompanied by virtually null-valued \(R_{RR}\).

(iii) Finally, when the chosen material is structurally right-handed but all components of \(\text{Re} \left[ \varepsilon(\mathbf{r}) \right]\) and \(\text{Re} \left[ \mu(\mathbf{r}) \right]\) are negative, the reflectance plots are exactly like that for case (ii).

The transmittances \(T_{LL} = |t_{LL}|^2\), etc. corroborate these observations. Clearly, the transformation (13) amounts to a change in structural handedness. This effect was observed in all computational results at every \(\lambda_0 \in [200, 2000] \text{ nm}\) tried; and it had been reported earlier [14] for the special case \(\theta = \chi = 0\).

A general proof for the handedness reversal of the circular Bragg phenomenon by virtue of a reversal of the signs of \(\text{Re} \left[ \varepsilon_{a,b,c} \right]\) and \(\text{Re} \left[ \mu_{a,b,c} \right]\) is as follows: Close examination of the matrix \(\left[ P(z) \right]\) reveals the following symmetries:

\[
\begin{align*}
\forall z \in [0, L], \quad \left[ P(z; \varepsilon(\mathbf{r}), \mu(\mathbf{r}); h, \psi) \right] &\to \left[ P(z; \varepsilon(\mathbf{r}), \mu(\mathbf{r}); -h, -\psi) \right]^*, \\
&\to \left[ P(z; -\varepsilon^*(\mathbf{r}), -\mu^*(\mathbf{r}); h, \pi + \psi) \right]^*.
\end{align*}
\]  
\tag{14, 15}

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where the $4 \times 4$ diagonal matrix $\bar{R} = \text{Diag} [1, -1, -1, 1]$ and the asterisk denotes the complex conjugate. Substitution of the symmetries (14) and (15) in (3) leads to the following identities for the tangential components of the electromagnetic fields:

$$\forall z \in [0, L], \left[ f(z; \imath_E (r), \mu (r); h, \psi) \right] = \left[ \bar{R} \left[ f(z; \imath_E (r), \mu (r); -h, -\psi) \right] \right]^{*}. \quad (16)$$

Application of the identity (16) to (9)-(11) suggests the relationship

$$\{ h \leftrightarrow -h, \psi \leftrightarrow -\psi \} \Rightarrow \{ a_L \leftrightarrow a_R, r_L \leftrightarrow r_R, t_L \leftrightarrow t_R \}; \quad (18)$$

while the identity (17) implies that

$$\begin{cases} \ Re [\epsilon_{a,b,c}] \rightarrow - Re [\epsilon_{a,b,c}] \\ \ Re [\mu_{a,b,c}] \rightarrow - Re [\mu_{a,b,c}] \end{cases} \Rightarrow \begin{cases} a_L \rightarrow -a_R^*, a_R \rightarrow -a_L^* \\ r_L \rightarrow -r_R^*, r_R \rightarrow -r_L^* \\ t_L \rightarrow -t_R^*, t_R \rightarrow -t_L^* \end{cases}. \quad (19)$$

The following two relationships thereby emerge:

$$\begin{cases} h \leftrightarrow -h, \psi \leftrightarrow -\psi \} \Rightarrow \begin{cases} r_{LL} \leftrightarrow r_{RR}, r_{LR} \leftrightarrow r_{RL} \\ t_{LL} \leftrightarrow t_{RR}, t_{LR} \leftrightarrow t_{RL} \end{cases}. \quad (20)$$

$$\begin{cases} \ Re [\epsilon_{a,b,c}] \rightarrow - Re [\epsilon_{a,b,c}] \\ \ Re [\mu_{a,b,c}] \rightarrow - Re [\mu_{a,b,c}] \end{cases} \Rightarrow \begin{cases} r_{LL} \rightarrow r_{RR}^*, r_{RR} \rightarrow r_{LL}^* \\ r_{LR} \rightarrow r_{RL}^*, r_{RL} \rightarrow r_{LR}^* \\ t_{LL} \rightarrow t_{RR}^*, t_{RR} \rightarrow t_{LL}^* \\ t_{LR} \rightarrow t_{RL}^*, t_{RL} \rightarrow t_{LR}^* \end{cases}. \quad (21)$$

The foregoing analysis thus proves that if (i) the signs of the real parts of the permittivity and permeability dyadics of a chiral ferrosmectic slab are reversed, and (ii) the transverse components of incident wave vector are also reversed, then the reflectances and the transmittances indicate that the structural handedness has been effectively reversed. Furthermore, a phase reversal of the reflection and transmission coefficients is also indicated.

With the relationships (20) and (21) involving all components of the permittivity and the permeability dyadics, no definite statement on handedness reversal can be extracted therefrom if not all components of the two dyadics suffer a change of sign in their real parts.

The reflection and transmission coefficients being weakly dependent on $\psi$ [5], calculations show that the handedness reversal of the circular Bragg phenomenon is evident, although approximately, even if the transverse components of the incident wave vector are not reversed. However, when $\overline{F(z)} \equiv [0, 0, z \leq L$, the reversal of the incident wave vector is not necessary for the handedness reversal of the circular Bragg phenomenon to be manifested exactly. This special condition holds for either normal incidence (i.e., $\theta = 0$) or cholesteric structure (i.e., $\chi = 0$), or both.

As a chiral ferrosmectic medium is continuously nonhomogeneous, general physical pictures of the circular Bragg phenomenon are difficult to develop, in contrast to their piecewise homogeneous counterparts [20]. Some understanding does become possible on analysis for normal incidence ($\kappa = 0$) and assuming the absence of dissipation. Then, the
eigenmodes inside a chiral ferrosmectic slab are either left or right elliptically polarized, with their respective vibration ellipses rotating along the z axis in accordance with the structural handedness of the material [21]. When $\epsilon_{a,b} > 0$ and $\mu_{a,b} > 0$, the direction of the phase velocity of a particular mode is the same as the (common) direction of energy transport. However, when $\epsilon_{a,b} < 0$ and $\mu_{a,b} < 0$, not only does the phase velocity reverse in direction, but the handedness of the vibration ellipse also reverses, while the direction of energy flow as well as the sense of rotation of the vibration ellipse remain unchanged. The reversal of all four modal handednesses amounts to an effective reversal of the structural handedness. As $[P(z)]$ is a holomorphic function of $\kappa$, the foregoing understanding would hold even for oblique incidence conditions, by virtue of analytic continuation [22]. For the same reason, the understanding should hold for weak dissipation too.

A consequence of the phase reversal of the reflection coefficients due to the transformation (13) are negative Goos-Hänchen shifts [19]. When conditions for total reflection prevail at a planar interface, a beam of finite width is seen to spring forward a certain distance on reflection. This distance is called the Goos-Hänchen shift. If the half-space $z \leq 0$ were occupied by an optically denser material than the chiral ferrosmectic slab, and if dissipation in that half-space were negligible, the phase reversal of the reflection coefficients would make the incident beam spring backwards on total reflection [23]. Causality, however, would not be violated.

4. Concluding remarks

When the real parts of the permittivity and the permeability dyadics of a chiral ferrosmectic slab are reversed in sign, the circular Bragg phenomenon displayed is proved here to suffer a change that indicates the effective reversal of structural handedness. Furthermore, reflection and transmission coefficients suffer phase reversal, which indicates the exhibition of negative Goos-Hänchen shifts when appropriate conditions prevail.

These conclusions are independent of the magnitudes of the real and imaginary parts of $\epsilon_{a,b,c}$ and $\mu_{a,b,c}$. Neither are the conclusions dependent of the variations of $\epsilon_{a,b,c}$ and $\mu_{a,b,c}$ with the angular frequency $\omega$. Therefore, the presented conclusions should hold without restrictions on anisotropy, dispersion and dissipation.