Meet Metamaterials

Akhlesh Lakhtakia

Department of Engineering Science & Mechanics
Pennsylvania State University

1600 hours, Thursday, August 24, 2006
Nittany Lion Inn, Penn State

A. Lakhtakia

Evolution

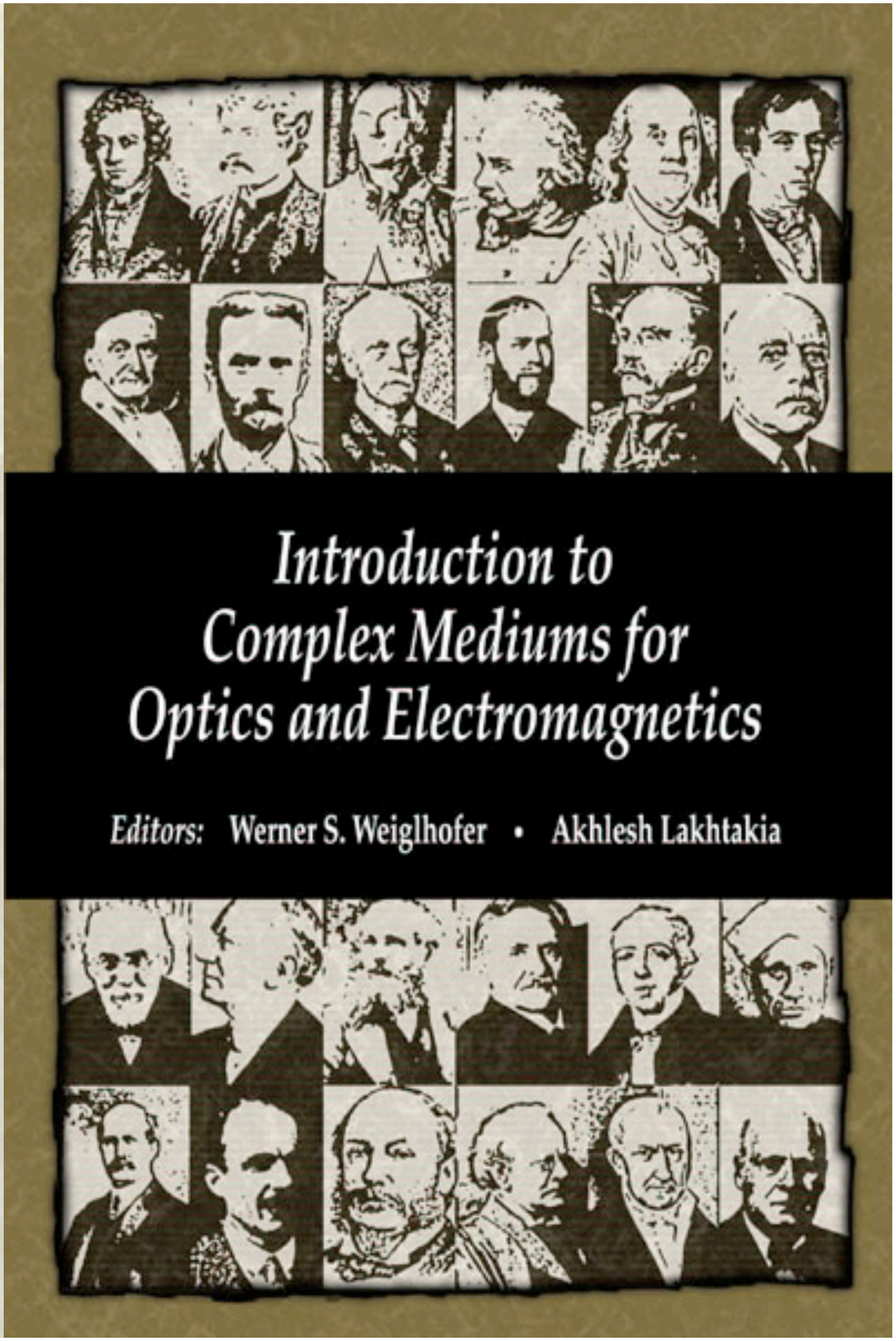
- Materials
- Function
- System Architecture

PENNSTATE

A. Lakhtakia

Metamaterials

Rodger Walser



A. Lakhtakia

Walser's Definition (2001/2)

- macroscopic composites having a manmade, three--dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation

Walser's Definition (2001/2)

- macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation

A. Lakhtakia

Walser's Definition (2001/2)

- macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation

A. Lakhtakia

Walser's Definition (2001/2)

- macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation

A. Lakhtakia

Walser's Definition (2001/2)

- macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation

Walser's Definition (2001/2)

- macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation

PENNSTATE

A. Lakhtakia

'Metamaterial'

– composite which exhibits properties:

  - $*$ not observed in constituents

  or

  - $*$ enhanced relative to properties of constituents

A. Lakhtakia

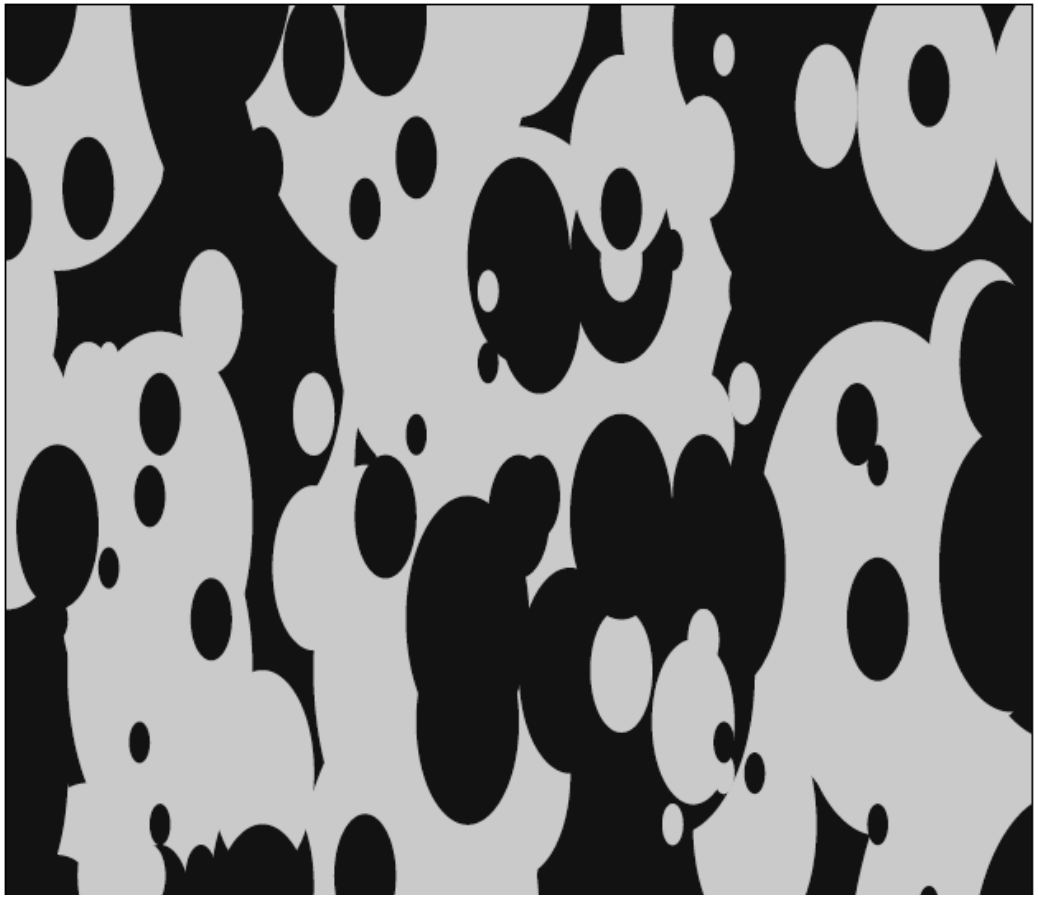
Conspirator-in-Chief:

Tom G. Mackay

School of Mathematics, University of Edinburgh

A. Lakhtakia

Particulate Composite Material with ellipsoidal inclusions



$\lambda \gg$ inclusion size

PENNSTATE

A. Lakhtakia

Overview

- Homogenization theory: SPFT

- Survey of HCM-based metamaterials:

1. Bianisotropy
2. Voigt wave propagation
3. Plane waves with negative phase velocity
4. Group velocity enhancement
5. Nonlinearity enhancement

Strong–property–fluctuation theory (SPFT)

- Higher–order distributional statistics
  - cf. Maxwell Garnett, Bruggeman
- Multi–scattering approach
- Provides iterative estimates of HCM constitutive parameters

A. Lakhtakia

Linear electromagnetic SPFT

- Constitutive relations:

$$\underline{D}(\underline{r}) = \underline{\underline{\epsilon}}(\underline{r}) \cdot \underline{E}(\underline{r}) + \underline{\underline{\xi}}(\underline{r}) \cdot \underline{H}(\underline{r})$$

$$\underline{B}(\underline{r}) = \underline{\underline{\zeta}}(\underline{r}) \cdot \underline{E}(\underline{r}) + \underline{\underline{\mu}}(\underline{r}) \cdot \underline{H}(\underline{r})$$

  – linear bianisotropic

  – frequency domain

1. Bianisotropy

- Bianisotropy through homogenization

- Example:

$$\underline{\underline{\mathbf{K}}}_a = \begin{bmatrix} \epsilon_0\epsilon_a \, \underline{\underline{I}} & i\sqrt{\epsilon_0\mu_0}\,\xi_a \, \underline{\underline{I}} \\ -i\sqrt{\epsilon_0\mu_0}\,\xi_a \, \underline{\underline{I}} & \mu_0\mu_a \, \underline{\underline{I}} \end{bmatrix}$$

chiral medium

$$\underline{\underline{\mathbf{K}}}_b = \begin{bmatrix} \epsilon_0 \, \underline{\underline{I}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \mu_0 \, \mathrm{diag}(\mu_b^x, \mu_b^y, \mu_b^z) \end{bmatrix}$$

biaxial magnetic medium

A. Lakhtakia

$$\longrightarrow \quad \underline{\underline{\mathbf{K}}}_{HCM}^{[n]} =$$

$$\begin{bmatrix} \epsilon_0 \begin{pmatrix} \epsilon^x_{HCM} & 0 & 0 \\ 0 & \epsilon^y_{HCM} & 0 \\ 0 & 0 & \epsilon^z_{HCM} \end{pmatrix} & i\sqrt{\epsilon_0\mu_0} \begin{pmatrix} \xi^x_{HCM} & 0 & 0 \\ 0 & \xi^y_{HCM} & 0 \\ 0 & 0 & \xi^z_{HCM} \end{pmatrix} \\ -i\sqrt{\epsilon_0\mu_0} \begin{pmatrix} \xi^x_{HCM} & 0 & 0 \\ 0 & \xi^y_{HCM} & 0 \\ 0 & 0 & \xi^z_{HCM} \end{pmatrix} & \mu_0 \begin{pmatrix} \mu^x_{HCM} & 0 & 0 \\ 0 & \mu^y_{HCM} & 0 \\ 0 & 0 & \mu^z_{HCM} \end{pmatrix} \end{bmatrix}$$

reciprocal biaxial bianisotropic medium

PENNSTATE

A. Lakhtakia

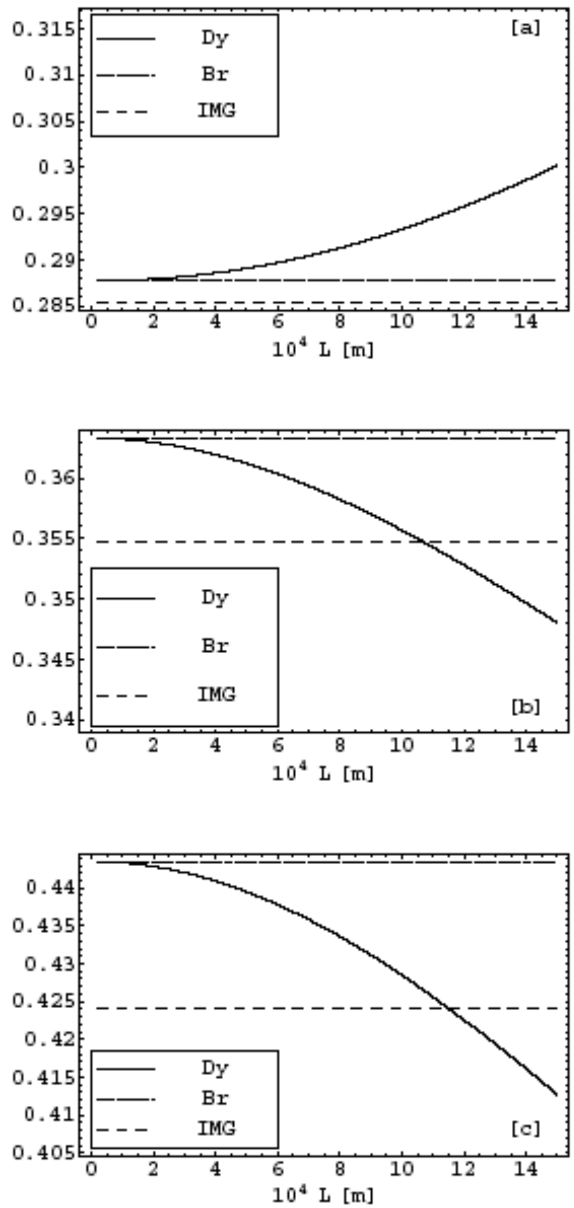


Figure 2: The real parts of (a) ξ^x_{Dy0}, (b) ξ^y_{Dy0} and (c) ξ^z_{Dy0} of a reciprocal bianisotropic composite plotted as functions of the correlation length L for $f_a = 0.3$. The L–independent values computed using the Bruggeman and Incremental Maxwell Garnett formalisms are also presented. See Section 7.2 for $\underline{\underline{\mathbf{K}}}_a$, $\underline{\underline{\mathbf{K}}}_b$, and $\underline{\underline{\tilde{\mathbf{K}}}}_{Dy}(\underline{0})$.

PENNSTATE

A. Lakhtakia

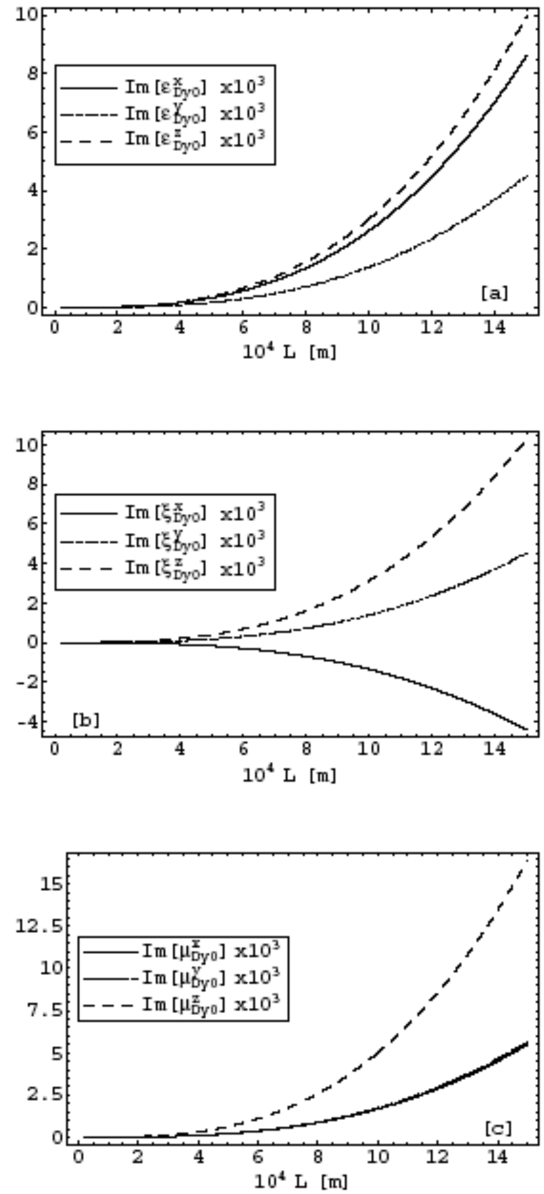


Figure 1: The imaginary parts of the effective constitutive scalars of a reciprocal bianisotropic composite plotted as functions of the correlation length L for $f_a = 0.3$. See Section 7.2 for $\underline{\underline{\mathbf{K}}}_a$, $\underline{\underline{\mathbf{K}}}_b$, and $\underline{\underline{\tilde{\mathbf{K}}}}_{Dy}(\underline{0})$.

2. Voigt wave propagation in HCMs

- Consider linear biaxial dielectric medium:

$$\left.\begin{aligned} \underline{D}(\underline{r}) &= \underline{\underline{\epsilon}} \bullet \underline{E}(\underline{r}) \\ \underline{B}(\underline{r}) &= \mu_0 \underline{H}(\underline{r}) \end{aligned}\right\}$$

with

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{pmatrix}$$

- Plane wave propagation along $\underline{\hat{z}}$ axis:

$$\underline{E}(\underline{r}) = \underline{\tilde{E}}\,\exp(ikz)$$

- Maxwell eqns $\longrightarrow$

$$\underline{\underline{P}} \cdot \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \end{pmatrix} = k^2 \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \end{pmatrix}$$

with

$$\underline{\underline{P}} = \frac{\omega^2 \mu_0}{\epsilon_{33}} \begin{pmatrix} \epsilon_{11}\epsilon_{33} - \epsilon_{13}\epsilon_{13} & \epsilon_{12}\epsilon_{33} - \epsilon_{13}\epsilon_{23} \\ \epsilon_{12}\epsilon_{33} - \epsilon_{23}\epsilon_{13} & \epsilon_{22}\epsilon_{33} - \epsilon_{23}\epsilon_{23} \end{pmatrix}$$

(A) Usual scenario – birefringence

- $\underline{\underline{P}}$:
  - 2 eigenvalues: $k_{\rm i}$ and $k_{\rm ii}$
  - 2 eigenvectors: $\begin{pmatrix} \tilde{E}_{x,\rm i} \\ \tilde{E}_{y,\rm i} \end{pmatrix}$ and $\begin{pmatrix} \tilde{E}_{x,\rm ii} \\ \tilde{E}_{y,\rm ii} \end{pmatrix}$
- General solution:

$$\begin{pmatrix} E_x(z) \\ E_y(z) \end{pmatrix} = C_{\rm i} \begin{pmatrix} \tilde{E}_{x,\rm i} \\ \tilde{E}_{y,\rm i} \end{pmatrix} \exp(ik_{\rm i}z) + C_{\rm ii} \begin{pmatrix} \tilde{E}_{x,\rm ii} \\ \tilde{E}_{y,\rm ii} \end{pmatrix} \exp(ik_{\rm ii}z)$$

A. Lakhtakia

(B) Anomalous scenario – Voigt wave

- $\underline{\underline{P}}$:
  - 1 eigenvalue: $k_\mathrm{i} = k_\mathrm{ii}$
  - 1 eigenvector: $\begin{pmatrix} \tilde{E}_{x,\mathrm{i}} \\ \tilde{E}_{y,\mathrm{i}} \end{pmatrix}$

- General solution:

$$\begin{pmatrix} E_x(z) \\ E_y(z) \end{pmatrix} = (C_\mathrm{i} + ik_\mathrm{i} z C_\mathrm{i}) \begin{pmatrix} \tilde{E}_{x,\mathrm{i}} \\ \tilde{E}_{y,\mathrm{i}} \end{pmatrix} \exp(ik_\mathrm{i} z)$$

W. Voigt (1902)
S. Pancharatnam (1958)

A. Lakhtakia

Conditions for Voigt wave propagation

- $V = 0$

  where

$$
\begin{aligned}
V = & \ \epsilon_{13}^4 + \epsilon_{23}^4 \\
& -2\epsilon_{23}\epsilon_{33}\left[2\epsilon_{12}\epsilon_{13} - (\epsilon_{11} - \epsilon_{22})\,\epsilon_{23}\right] \\
& + \left[(\epsilon_{11} - \epsilon_{22})^2 + 4\epsilon_{12}^2\right]\epsilon_{33}^2 \\
& +2\epsilon_{13}\Big\{\epsilon_{23}^2\epsilon_{13} \\
& -\left[2\epsilon_{12}\epsilon_{23} + (\epsilon_{11} - \epsilon_{22})\,\epsilon_{13}\right]\epsilon_{33}\Big\}
\end{aligned}
$$

- $W \neq 0$

  where

$$W = \epsilon_{12}\epsilon_{33} - \epsilon_{13}\epsilon_{23}$$

A. Lakhtakia

Voigt waves in biaxial HCMs

- Consider 2 uniaxial component phases (spherical particulate geometry):

$$\left.\begin{aligned} \underline{\underline{\epsilon}}^a &= \underline{\underline{R}}_z(\varphi) \cdot [\mathrm{diag}\,(\epsilon_x^a, \epsilon^a, \epsilon^a)] \cdot \underline{\underline{R}}_z^T(\varphi) \\ \underline{\underline{\epsilon}}^b &= \mathrm{diag}\left(\epsilon_x^b, \epsilon^b, \epsilon^b\right) \end{aligned}\right\}$$

where

$$\underline{\underline{R}}_z(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} \underline{\underline{\tilde{\epsilon}}}^{HCM} &= \underline{\underline{R}}_z(\psi^R) \cdot \left[\mathrm{diag}\left(\epsilon_x^R, \epsilon_y^R, \epsilon_z^R\right)\right] \cdot \underline{\underline{R}}_z^T(\psi^R) \\ &\quad + i\,\underline{\underline{R}}_z(\psi^I) \cdot \left[\mathrm{diag}\left(\epsilon_x^I, \epsilon_y^I, \epsilon_z^I\right)\right] \cdot \underline{\underline{R}}_z^T(\psi^I) \end{aligned}$$

A. Lakhtakia

- Explore possibility of

$$\left.\begin{array}{l} V = 0 \\ W \neq 0 \end{array}\right\} \quad \text{Voigt wave propagation}$$

for

$$\begin{aligned}
\underline{\underline{\epsilon}}^{HCM}(\alpha,\beta,\gamma) &= \underline{\underline{R}}_z(\gamma)\cdot\underline{\underline{R}}_y(\beta)\cdot\underline{\underline{R}}_z(\alpha)\cdot\underline{\underline{\tilde{\epsilon}}}^{HCM} \\
&\quad \cdot\underline{\underline{R}}_z^T(\alpha)\cdot\underline{\underline{R}}_y^T(\beta)\cdot\underline{\underline{R}}_z^T(\gamma) \\
&= \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{pmatrix}
\end{aligned}$$

where

$$\underline{\underline{R}}_y(\beta) = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$

PENNSTATE
A. Lakhtakia

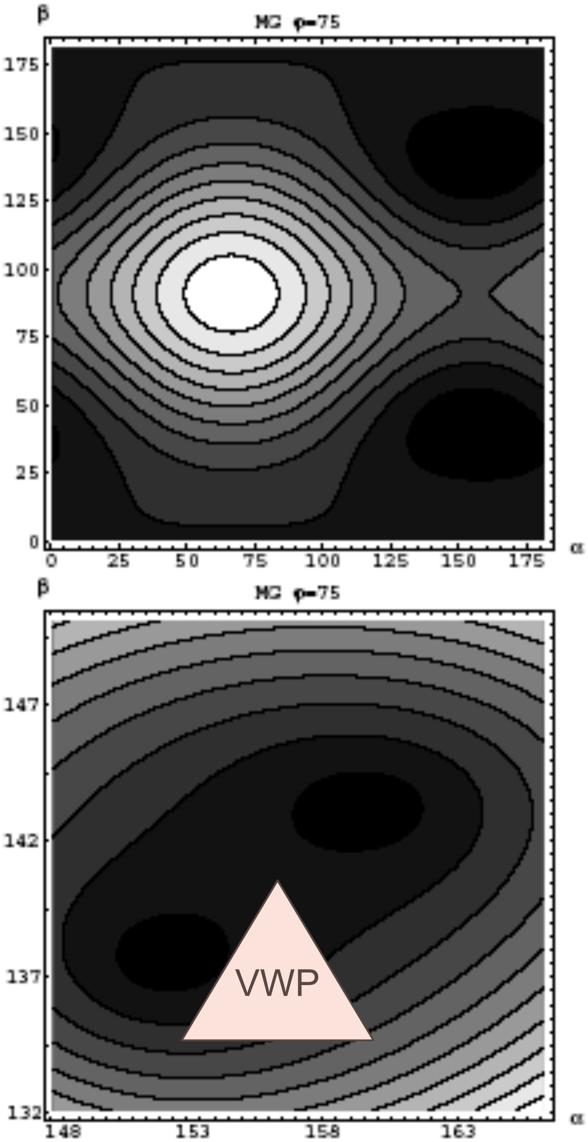


Figure 5: The calculated values of $|V|$ plotted against α and β (in degrees) for the orientation angle $\varphi = 75°$ of component phase a. The left side graphs emerge from the Maxwell Garnett formalism, whereas the right side are due to the Bruggeman formalism. The lower contour plots each show a magnified neighbourhood of a pair of zeros of V.

PENNSTATE
A. Lakhtakia

3. Planewaves with negative phase velocity

- Suppose isotropic dielectric–magnetic medium:

(a) Nondissipative

$$\epsilon(\omega) < 0\,, \qquad \mu(\omega) < 0$$

or:

(b) Dissipative

$$\frac{\mathrm{Re}\{\epsilon(\omega)\}}{\mathrm{Im}\{\epsilon(\omega)\}} + \frac{\mathrm{Re}\{\mu(\omega)\}}{\mathrm{Im}\{\mu(\omega)\}} < 0$$

Then:

- wavevector $\underline{k}$ ↑

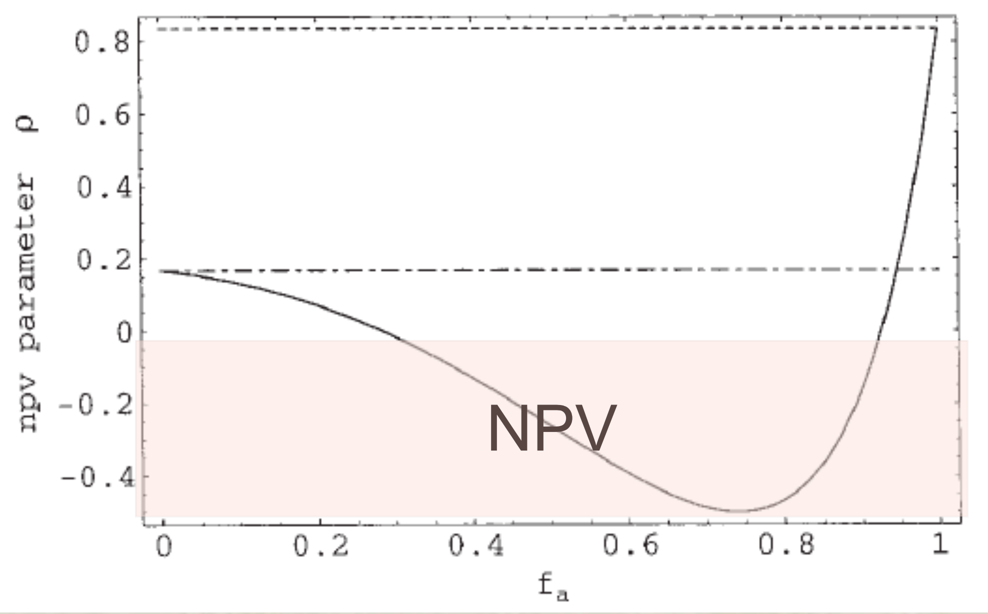
  Poynting $\underline{P} = \frac{1}{2}\mathrm{Re}\{\underline{E} \times \underline{H}^*\}$ ↓

- i.e., 'negative phase velocity' medium

NPV metamaterial arising from non–NPV constituents

Isotropic dielectric–magnetic mediums:

$$\varepsilon_a = -6 + 0.9i, \quad \mu_a = 1.5 + 0.2i$$
$$\varepsilon_b = -1.5 + i, \quad \mu_b = 2 + 1.2i$$



A. Lakhtakia

NPV metamaterial arising from non–NPV constituents

Anisotropic/bianisotropic/chiral mediums:

More scope for NPV as more wavenumbers

Example: Faraday chiral medium (FCM)

$$\underline{D}(\underline{r}) = \underline{\underline{\epsilon}} \cdot \underline{E}(\underline{r}) + \underline{\underline{\xi}} \cdot \underline{H}(\underline{r})$$
$$\underline{B}(\underline{r}) = -\underline{\underline{\xi}} \cdot \underline{E}(\underline{r}) + \underline{\underline{\mu}} \cdot \underline{H}(\underline{r})$$

$$\underline{\underline{\epsilon}} = \epsilon\,\underline{\underline{I}} - i\epsilon_g\,\hat{\underline{z}} \times \underline{\underline{I}} + (\,\epsilon_z - \epsilon\,)\;\hat{\underline{z}}\,\hat{\underline{z}}$$

$$\underline{\underline{\xi}} = i\left[\xi\,\underline{\underline{I}} - i\xi_g\,\hat{\underline{z}} \times \underline{\underline{I}} + (\,\xi_z - \xi\,)\;\hat{\underline{z}}\,\hat{\underline{z}}\right]$$

$$\underline{\underline{\mu}} = \mu\,\underline{\underline{I}} - i\mu_g\,\hat{\underline{z}} \times \underline{\underline{I}} + (\,\mu_z - \mu\,)\;\hat{\underline{z}}\,\hat{\underline{z}}$$

- Faraday chiral medium as HCM arising from non–NPV components:
  - Isotropic chiral phase a

$$\underline{D} = \epsilon_0 \epsilon^a \, \underline{E} + i\sqrt{\epsilon_0 \mu_0}\, \xi^a \, \underline{H}$$
$$\underline{B} = -i\sqrt{\epsilon_0 \mu_0}\, \xi^a \, \underline{E} + \mu_0 \mu^a \, \underline{H}$$

  - Magnetically–biased ferrite phase b

$$\underline{D} = \epsilon_0 \epsilon^b \, \underline{E}$$
$$\underline{B} = \mu_0 \left[\mu^b \, \underline{\underline{I}} - i \mu_g^b \, \hat{\underline{z}} \times \underline{\underline{I}} + \left(\mu_z^b - \mu^b \right) \hat{\underline{z}}\, \hat{\underline{z}} \right] \bullet \underline{H}$$

PHYSICAL REVIEW E **69**, 026602 (2004)

Plane waves with negative phase velocity in Faraday chiral mediums

Tom G. Mackay*
School of Mathematics, James Clerk Maxwell Building, The King's Buildings, University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom

Akhlesh Lakhtakia†
CATMAS — Computational and Theoretical Materials Sciences Group, Department of Engineering Science and Mechanics, Pennsylvania State University, University Park, Pennsylvania 16802-6812, USA

(Received 18 July 2003; published 10 February 2004)

The propagation of plane waves in a Faraday chiral medium is investigated. Conditions for the phase velocity to be directed opposite to the direction of power flow are derived for propagation in an arbitrary direction; simplified conditions which apply to propagation parallel to the distinguished axis are also established. These negative phase-velocity conditions are explored numerically using a representative Faraday chiral medium, arising from the homogenization of an isotropic chiral medium and a magnetically biased ferrite. It is demonstrated that the phase velocity may be directed opposite to power flow, provided that the gyrotropic parameter of the ferrite component medium is sufficiently large compared with the corresponding nongyrotropic permeability parameters.

DOI: 10.1103/PhysRevE.69.026602 PACS number(s): 41.20.Jb, 42.25.Bs, 83.80.Ab

A. Lakhtakia

4. Group velocity enhancement

- Group velocity of wavepacket:
  (isotropic dielectric)

$$v = \frac{c}{n(\omega) + \omega \frac{dn}{d\omega}} \Bigg|_{\omega(k_{avg})} \qquad \left(c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \right)$$

- HCM refractive index from Bruggeman:

$$f_a \frac{n_a^2 - n_{Br}^2}{n_a^2 + 2n_{Br}^2} + f_b \frac{n_b^2 - n_{Br}^2}{n_b^2 + 2n_{Br}^2} = 0$$

- Homogenization of components with
  - phase a: large n_a but small $\frac{dn_a}{d\omega}$
  - phase b: small n_b but large $\frac{dn_b}{d\omega}$

A. Lakhtakia

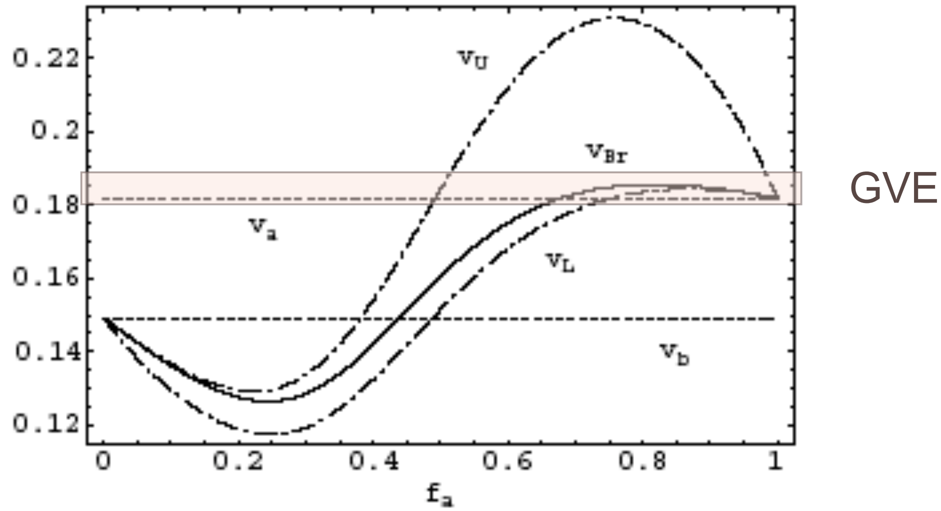


Figure 12: The group velocity v_{Br} (solid line) and its upper and lower bounds (broken dashed lines), along with the component phase group velocities v_a and v_b (broken dashed lines), plotted as functions of the volume fraction f_a. All velocities are normalized with respect to c.

A. Lakhtakia

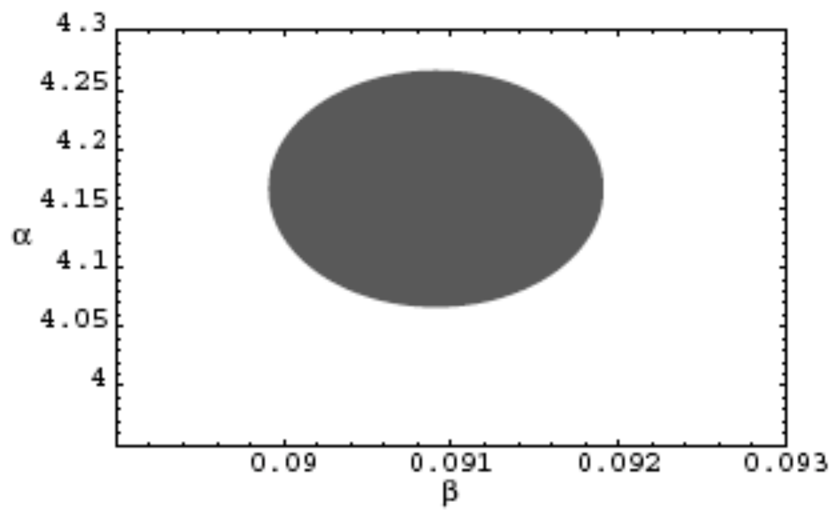


Figure 13: The shaded region indicates the region of $\alpha = \frac{n_a}{n_b}$ and $\beta = \frac{dn_a}{d\omega} / \frac{dn_b}{d\omega}$ phase space where $v_{Br} > v_a$ and $v_{Br} > v_b$. Values of the component phase a parameters are fixed at $n_a = 5$, $\frac{dn_a}{d\omega} = 0.5/\omega$ and $f_a = 0.8$.

5. Nonlinearity enhancement

- Homogenization of isotropic dielectric components:
  - weakly nonlinear phase a:

$$\epsilon_a = \epsilon_{a0} + \chi_a \, | \underline{E}_a |^2$$

with $| \epsilon_{a0} | \gg | \chi_a | \, | \underline{E}_a |^2$

  - linear phase b: ϵ_b
  - ellipsoidal particles: $\underline{\underline{U}} = \text{diag}(U_x, U_y, U_z)$

A. Lakhtakia

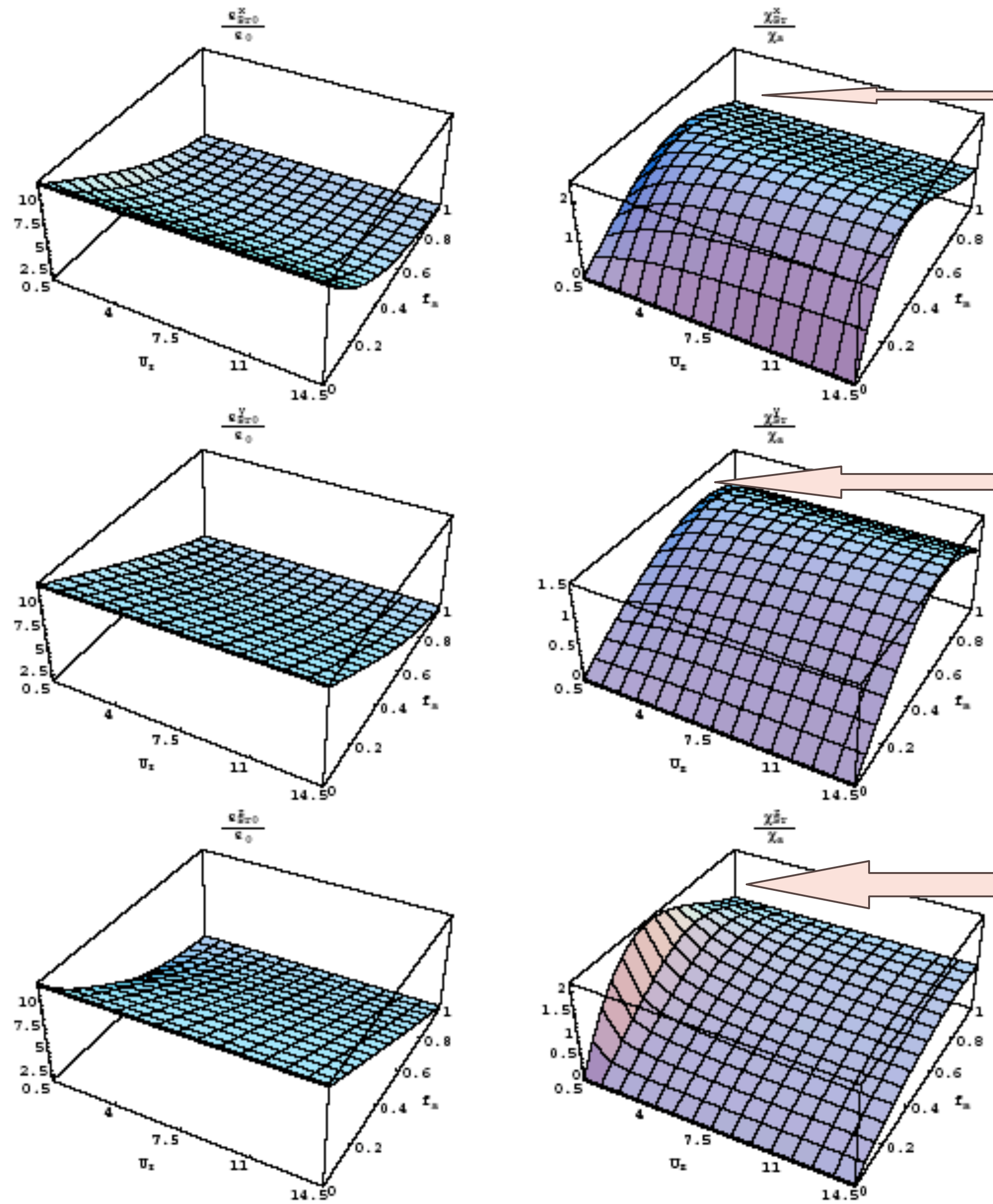
- $\longrightarrow$ weakly nonlinear anisotropic HCM:

$$\begin{aligned}\underline{\underline{\epsilon}}_{\text{HCM}} &= \underline{\underline{\epsilon}}_{\text{HCM0}} + \underline{\underline{\chi}}_{\text{HCM}}\,|\underline{E}_{\text{HCM}}|^2\\ &= \text{diag}\left(\epsilon^x_{\text{HCM0}},\,\epsilon^y_{\text{HCM0}},\,\epsilon^z_{\text{HCM0}}\right)\\ &\quad+\text{diag}\left(\chi^x_{\text{HCM}},\,\chi^y_{\text{HCM}},\,\chi^z_{\text{HCM}}\right)\,|\underline{E}_{\text{HCM}}|^2\end{aligned}$$

PENNSTATE

1855

A. Lakhtakia



NLE

NLE

NLE

Figure 14: HCM relative linear permittivity and nonlinear susceptibility parameters calculated using the Bruggeman homogenization formalism. Component phase parameter values: $\epsilon_{a0} = 2\epsilon_0$, $\chi_a = 9.07571 \times 10^{-12}\epsilon_0\,\mathrm{m}^2\mathrm{V}^{-2}$, $\epsilon_b \equiv \epsilon_{b0} = 12\epsilon_0$, $U_x = 1$ and $U_y = 3$.

Summary

- HCMs conceptualized which exhibit properties:
  - not observed in constituents

  or

  - enhanced relative to properties of constituents

i.e., homogenization an important concept in design of metamaterials