Would Brewster recognize today's Brewster angle?

By Akhlesh Lakhtakia

The oldest and simplest way of treating light is in terms of geometrical optics, and, over the centuries, it has sufficed to explain most practical applications. Perhaps the first systematic and intensive studies were conducted by the Arab mathematician and optician Abu Ali Al-Hasan Ibn Al-Haytham (b.*c.* 965 Basra, Iraq; d. 1039 Cairo, Egypt), whose work has been commemorated on the Pakistani stamp shown on the cover. In his treatise translated into Latin in 1270, Alhazen, as he became known to medieval Europe, theorized on various issues in optics: reflection, refraction, mirrors and lenses, aberrations, rainbows, and atmospheric refraction. Most notably, he correctly deduced that light from the object seen comes to the eye.

Later opticians busied themselves with the construction and improvements of optical instruments and did not bother about the nature of light. The 17th century, however, saw great strides made in the latter direction. René Déscartes (1596–1650) and Pierre de Fermat (1601–1665) provided a mathematical framework for ray optics; Willebrod van Snel van Royen (1580–1626) codified the laws of reflection and refraction; while Erasmus Bartholinus (1625–1698), a Danish mathematician and doctor, discovered the delights of the calcite rhomb, or Iceland spar, which made many other discoveries possible. Toward the end of the 17th century, Christiaan Huygens (1629–1695) proposed the wave-theory, while Sir Isaac Newton (1643–1727) championed the cause of the particulate nature of light. With the further discovery of the interference and diffraction phenomena, the science of optics became firmly established.

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In the same work—*Traite dé la Lumière* (publ. 1690)—in which he set forth what is now universally known as Huygens's principle, Huygens gave a brief account of "a wonderful phenomenon which I discovered after I had written all that has gone before."[1] While experimenting with Iceland spar, he observed[2] that a ray of light was divided into separate rays of equal intensity after passing through the crystal. The only condition for the separation of light not to occur was if light traversed the crystal parallel to the crystallographic axis. Subsequent experiments of Huygens led us to believe that he had discovered the polarization of light, but his discovery amounted to very little because he remained unaware of the true nature of light.

In 1801, after various involvements in the Napoleonic wars, an Army engineer named Etienne-Louis Malus

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(1775–1812) joined the faculty of the École Polytechnique in Paris. As Biot related later, Malus recognized the phenomenon of polarization by reflection by observing through a crystal of Iceland spar the sunlight reflected from the windows of the Luxembourg.[3] The "power of changing the character of light and of giving it a new property, which it carries with it," Malus wrote,[4] "is not peculiar to Iceland spar: I have found it in all known substances which give double images." In subsequently coining the term *polarization*, Malus employed the analogy of magnetic poles.

Brewster and the polarizing angle

Iceland spar figured in once again—prior to the introduction of the prisms of Nicol, Wollaston, and Rochon, and the 20th century invention of the plastic polarizer sheets by Edwin Land—in the voluminous investigations of Sir David Brewster, a Scottish clergyman-turned-physicist who lived during the years 1781 and 1868. The transverse-wave theory of light[5] had yet to be established when Brewster commenced his studies on the reflection of unpolarized light by planar dielectric-dielectric interfaces. By extensive experimentation, he showed that when the sum of the angles of incidence and refraction equals $\pi/2$, the reflected light is linearly polarized, but the transmitted field has both parallel- and perpendicularly-polarized components; such an angle of incidence was called the *polarizing* angle. In a celebrated paper,[6] Brewster's experiments caused him to set forth the law that now bears his name. When the medium of refraction is slightly absorbing, then a psuedo-polarizing angle[7] is obtained.

In view of Professor Barr's recent biographical sketch[8] of Sir David Brewster, who has been immortalized in optics also by the Brewster fringes, the following question is of relevance: Would Brewster recognize the manner of introducing the Brewster angle in many modern textbooks? Let us explore some recent textbooks on this issue, it being understood that all of these texts deal with the interfaces of homogeneous, isotropic, lossless dielectric media.

Jackson:[9] . . . for polarization parallel to the plane of incidence, there is an angle of incidence, called *Brewster's angle*, for which there is no reflected wave.

Kraus:[10] . . . it is possible to find an incident angle so that . . . the wave is *totally transmitted* . . . This angle, called the *Brewster angle*, . . .

van Bladel:[11] . . . there is an angle, called the *Brewster angle*, for which a wave polarized parallel to the plane of incidence has zero reflection coefficient.

Shen and **Kong:**[12] . . . For parallel polarization, there is always an angle . . . such that the wave is totally transmitted.

Would Brewster recognize the manner of introducing the Brewster angle in many modern textbooks?

Bohren and **Huffman:**[13] . . . there is an angle of incidence for which [the reflection coefficient] $R_{\parallel} = 0$. This angle is called the *polarizing or Brewster* angle.

Harrington:[14] . . . There is usually an angle of total transmission . . . called the *polarizing angle or Brewster angle.*

Even in the celebrated opus of Shurcliff,[15] the Brewster angle has been introduced by equating $R_{\parallel}$ (his ρ_0) to zero. An informal survey of my colleagues revealed that this understanding of the Brewster angle is widespread and was not simply confined to me alone.

In these and other popularly used books, an additional piece of information follows: if an unpolarized planewave is incident on the interface at the concerned angle, then the reflected wave is linearly polarized. This fact seems to have been the predominant, if not the only, one used to introduce the Brewster angle in books[16–19] written prior to the Second World War. Classics in optics generally tend to introduce the Brewster angle in this fashion, though there are exceptions:

Born and **Wolf:** [20] . . . consequently ($R_{\parallel} = 0$) In this case . . . it follows . . . that $\tan\theta_i = n$. The angle θ_i . . . is called *the polarizing or Brewster* angle.

Strong:[21] . . . The angle of incidence which makes $(i + r) = \pi/2$ is called Brewster's angle . . . This polarization by reflection at Brewster's angle . . .

Houston:[22] . . . Brewster showed that the tangent of the polarising angle was equal to the index of refraction of the medium in question.

Jenkins and **White:**[23] It was Brewster who first discovered that at this polarizing angle . . . the reflected and refracted rays are just 90° apart.

Sommerfeld:[24] . . . The R_s curve crosses the abscissa . . . [at the] . . . angle of polarization . . . Since R_s vanishes at this angle, . . .

Two definitions emerge

Two definitions of the Brewster angle are thus apparent.[25] The first one is that of a *zero-reflection angle*, for which a parallel-polarized plane wave is totally transmitted. The second one is that of a *polarizing angle*, for which an unpolarized plane wave is reflected as a linearly-polarized plane wave. Brewster, following upon the work of Malus, obtained his angle by examining experimentally measured values of the *polarizing* angle. Since Fresnel[26] had yet to publish his seminal work on polarization in 1812—he did that in 1822—Brewster could not have thought in terms of zero reflection coefficients. And after Fresnel, no particular need arose to make the distinction either, as we shall presently see.

If $E_{i\perp}$ and $E_{i\parallel}$ are the amplitudes of the parallel- and perpendicularly-polarized components of the incident plane wave, and $E_{r\perp}$ and $E_{r\parallel}$ refer to that of the reflected wave, then a Jones matrix can be defined via

$$E_{r\perp} = R_{11}E_{i\perp} + R_{12}E_{i\parallel}; \quad E_{r\parallel} = R_{21}E_{i\perp} + R_{22}E_{i\parallel}. \quad (1)$$

It can be shown[27] that the condition for obtaining a linearly-polarized plane wave is given by

$$R_{11}(\theta_{inc})R_{22}(\theta_{inc}) - R_{12}(\theta_{inc})R_{21}(\theta_{inc}) = 0, \quad (2)$$

in which θ_{inc} is the angle of incidence with respect to the normal to the interface. If such an angle exists, then it is to be referred to as the *polarizing* angle θ_p. On the other hand, *zero-reflection* angles (θ_{znm}) can be conveniently defined by

$$R_{nm}(\theta_{inc} = \theta_{znm}) = 0; \; n = 1, 2; \; m = 1, 2. \quad (3)$$

Let the medium of incidence and reflection have the real isotropic constitutive constants ε and μ, while the similar medium of transmission have the constants $\varepsilon_t = \varepsilon_r\varepsilon$ and $\mu_t = \mu_r\mu$. Then, $R_{12} = R_{21} = 0$; and using standard analysis,[28] it can be shown that

$$\tan^2\theta_{z11} = \mu_r(\mu_r - \varepsilon_r)/(\mu_r\varepsilon_r - 1), \quad (4a)$$
$$\tan^2\underset{\theta}{\cancel{\mu}}_{z22} = \varepsilon_r(\varepsilon_r - \mu_r)/(\mu_r\varepsilon_r - 1), \quad (4b)$$

Equation (4a) gives a real angle θ_{z11} if $\mu_r > \varepsilon_r$, while (4b) gives a real angle θ_{r22} if $\mu_r < \varepsilon_r$. Under these conditions, (2) simplifies to

$$R_{11}(\theta_{inc})R_{22}(\theta_{inc}) = 0, \quad (5)$$

so that θ_{z11} or θ_{z22}, as the case may be, is also the *polarizing* angle. This is probably the reason for abandoning Brewster's own definition of the Brewster angle as a *polarizing* angle, and defining the Brewster angle straightaway as a *zero-reflection* angle. It has been pointed out that the development of lasers may have some bearing on this change of definition: Brewster angle windows are extensively used to provide a low loss method of letting parallel-polarized light exit from the laser.

Just as an anisotropic (birefringent) crystal assisted in the development of Brewster's investigation, circularly birefringent materials[29,30]—also called optically active or chiral—come to our aid in a reappraisal of the Brewster angle. Optically active media are well-known to organic chemists, and recent years have witnessed considerable progress in the electromagnetic field theory pertinent to such materials.[31,32] Although chiral media can be modelled by several different, and equivalent, constitutive equations,[31] use will be made here of the following set given by Post:[33]

$$\mathbf{D} = \varepsilon_t\mathbf{E} + i\gamma\mathbf{B}; \qquad \mathbf{H} = (1/\mu_t)\mathbf{B} + i\gamma\mathbf{E}, \quad (6)$$

γ being the chirality parameter. Consider the reflection of an unpolarized planewave at the planar interface of a dielectric material with a homogeneous chiral medium. The elements of the Jones matrix (1) have been worked out to be[34]

$$\Delta R_{11} = \cos\theta_{inc}\,(1\text{-}g^2)(\cos\theta_1 + \cos\theta_2) + 2g(\cos^2\theta_{inc} - \cos\theta_1\cos\theta_2), \quad (7a)$$
$$\Delta R_{22} = \cos\theta_{inc}\,(1\text{-}g^2)(\cos\theta_1 + \cos\theta_2) - 2g(\cos^2\theta_{inc} - \cos\theta_1\cos\theta_2), \quad (7b)$$
$$\Delta R_{21} = \Delta R_{12} = -2ig\,\cos\theta_{inc}(\cos\theta_1 - \cos\theta_2), \quad (7c)$$

in which

$$\Delta = \cos\theta_{inc}\,(1+g^2)(\cos\theta_1 + \cos\theta_2) + 2g(\cos^2\theta_{inc} + \cos\theta_1\cos\theta_2), \quad (8a)$$
$$g^2 = (\mu/\varepsilon)\gamma^2 + \varepsilon_r/\mu_r; \qquad k = +\,\omega\surd(\mu\varepsilon), \quad (8b)$$
$$\sin\theta_1 = (k/h_1)\sin\theta_{inc}; \quad \sin\theta_2 = (k/h_2)\sin\theta_{inc}, \quad (8c)$$
$$h_1 = \omega\mu_t\gamma + \surd(\omega^2\mu_t^2\gamma^2 + \omega^2\mu_t\varepsilon_t);$$
$$h_2 = -\omega\mu_t\gamma + \surd(\omega^2\mu_t^2\gamma^2 + \omega^2\mu_t\varepsilon_t). \quad (8d)$$

Unless $\gamma = 0$, it is impossible to have $\cos\theta_1 = \cos\theta_2$ if $\theta_{inc} \neq 0$. Thus, from (7c) it is easily seen that

$$\theta_{z21} = \theta_{z12} = 0 \text{ or } \pi/2, \quad (9)$$

which is a trivial result. Next, θ_{z11} and θ_{z22} can be obtained by equating the right sides of (7a) and (7b), respectively, to zero. And polarizing angles θ_p can be calculated by substituting (7a,b,c) into (2) and solving the resulting transcendental equation.

Can an angle of incidence simultaneously satisfy the condition $R_{22} = 0$ (or $R_{11} = 0$) and the transcendental equation (2) for the dielectric-chiral interface? Let θ_o be such that

$$R_{22}(\theta_{inc} = \theta_o) = 0; \quad (10a)$$

parenthetically, it should be noted that $R_{22}(\theta_{inc} = 0) = 1\text{-}g)/(1+g)$ and $R_{22}(\theta_{inc} = \pi/2) = 1$. Then, in order that it also be a polarizing angle, it follows that

$$R_{12}(\theta_{inc} = \theta_o)\, R_{21}(\theta_{inc} = \theta_o) = 0. \quad (10b)$$

But (10b) demands that (i) either $\cos\theta_1 = \cos\theta_2$ (which is not possible for chiral media unless $\theta_o = 0$), or (ii) $\theta_o = \pi/2$!

. . . as we venture into the territory of novel materials it matters to have a proper definition of the Brewster angle, consistent with current understanding, available.

That confusion has prevailed about the Brewster angle for some time also becomes apparent from a treatment of a planar vacuum/chiral interface dating back to 1960. Bokut' and Fedorov[35] modelled their non-magnetic, non-reciprocal[36] chiral medium by $\mathbf{D} = \epsilon_r \epsilon_o [\mathbf{E} + \alpha \nabla \times \mathbf{E}]$ and $\mathbf{B} = \mu_o \mathbf{H}$; here, ϵ_o and μ_o are the permittivity and the permeability, respectively, of the vacuum. They obtained a solution θ_p of (2) for this case, observing that "the reflected light is linearly polarized with its plane of oscillation normal to the plane of incidence." Further, they went on to state that θ_p is "very close to the Brewster angle of the medium." Evidently, they reasoned that Brewster angle *must* be defined equal to $\tan^{-1}(\sqrt{\epsilon_r})$!

Thus, a case involving isotropic materials exists in which the *polarizing* angle is not the *zero-reflection* angle. Which one of them is the *Brewster* angle? Brewster himself, if his original paper is any guide, would have preferred the *polarizing* angle. But generations of scientists now have been first exposed to the Brewster angle as the *zero-reflection* angle. The distinction is of no importance for dielectric-dielectric interfaces, but as we venture into the territory of novel materials it matters to have a proper definition of the Brewster angle, consistent with current understanding, available. Perhaps the Optical Society of America will take the lead!

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