On extending the Brewster law at planar interfaces

A. Lakhtakia
Department of Engineering Science and Mechanics, The Pennsylvania State University, USA

Received November 13, 1989

Abstract

On extending the Brewster law at planar interfaces. The reflection of planewaves at planar chiral-uniaxial interfaces has been examined in order to broaden the concept of the Brewster law into what may be termed as the Brewster reflection de-correlation condition.

Inhalt


Introduction

In 1815, Sir David Brewster described [1] his experiments on the reflection of unpolarised light from planar dielectric-dielectric interfaces. Data collected by him gave rise to what is now called the Brewster angle, and was condenscd by him into the Brewster law. Modern textbooks tend to give an un-Brewsterian definition of the Brewster law [2], which is more faithfully stated for dielectric-dielectric interfaces as: If unpolarized light is incident at this angle, the reflected light is plane-polarized. It is the purpose of this communication to broaden the concept of the Brewster angle into what may be termed as the Brewster reflection de-correlation condition. This will be done by examining the reflection of plane waves at planar chiral-uniaxial interfaces.

Theoretical Development

Consider the interface \( z = 0 \): a homogeneous, lossless, uniaxial dielectric medium occupies the half-space \( z \geq 0 \); while the half-space \( z \leq 0 \) is filled with an isotropic, homogeneous, lossless, chiral medium.

The chiral medium, characterized by [3]

\[
D = \varepsilon [E + \beta V \times E]; \quad B = \mu [H + \beta V \times H],
\]

(1)
is circularly birefringent. Thus, the fields in the region \( z \leq 0 \) are circularly polarised, and may be represented using the vectors [3]

\[
Q_1 = A_1 [e_y + i(\delta y_1 e_x + \kappa e_z)/\gamma_1] \exp [i(k x + \delta_1 z)]
+ B_1 [e_y + i(\delta y_1 e_x + \kappa e_z)/\gamma_1] \exp [i(k x - \delta_1 z)]; \quad z \leq 0,
\]

(2a)

\[
Q_2 = A_2 [e_y + i(\delta y_2 e_x - \kappa e_z)/\gamma_2] \exp [i(k x + \delta_2 z)]
+ B_2 [e_y + i(\delta y_2 e_x + \kappa e_z)/\gamma_2] \exp [i(k x - \delta_2 z)]; \quad z \leq 0.
\]

(2b)

Here, the wavenumbers are given by \( \gamma_1 = k/(1 - k \beta) \) and \( \gamma_2 = k/(1 + k \beta) \); \( k = \omega \sqrt{\varepsilon / \mu} \) is merely a shorthand and notation; while \( \delta_1 = + \sqrt{\gamma_1^2 - \kappa^2} \) and \( \delta_2 = + \sqrt{\gamma_2^2 - \kappa^2} \). The coefficients \( A_1 \) and \( A_2 \) represent plane waves incident on the interface, while \( B_1 \) and \( B_2 \) denote the plane waves reflected off into the chiral half-space. The electromagnetic fields in this region are given by

\[
E = Q_1 - i \eta Q_2, \quad H = Q_2 - (i/\eta) Q_1; \quad z \leq 0,
\]

(3)

with \( \eta = \sqrt{\mu/\varepsilon} \). An \exp [i \omega t] time-dependence has been assumed, while \( \kappa \) is the horizontal wavenumber required by Snell’s law to satisfy the phase-matching condition at the interface \( z = 0 \); and \( e_x \), etc., are the unit Cartesian vectors.

The constitutive relations for the uniaxial medium are specified as [4]

\[
D = \varepsilon_{xx} E + (\varepsilon_{yy} - \varepsilon_{xx}) e(c \cdot E); \quad B = \mu H, \quad z \geq 0
\]

(4)
in which the optic axis is represented by the unit vector \( e \), in all generality [5], as

\[
e = e_x \sin \xi + e_z \cos \xi, 0^\circ \leq \xi \leq 180^\circ.
\]

(5)

It is well known that the planewaves in the uniaxial medium are of the ordinary and the extraordinary types. Thus, an appropriate representation of the plane waves in this half-space can be set down as

\[
E_x = C_1 \exp [i(k x + \delta_{2w} z)]
+ D_2 \exp [i(k x - \delta_{2w} - z)],
\]

(6a)

\[
E_y = C_1 \exp [i(k x + \delta_{1w} z)]
+ D_1 \exp [i(k x - \delta_{1w} - z)],
\]

(6b)
\[ H_x = J_{-1} \left[ -C_1 \exp[i(kx + \delta_{1u})] \\
+ D_1 \exp[i(kx - \delta_{1u})] \right], \tag{6c} \]
\[ H_y = J_{-2} \left[ C_2 \exp[i(kx + \delta_{2u})] \\
- D_2 \exp[i(kx - \delta_{2u})] \right], \tag{6d} \]
\[ H_z = \left( \kappa/\omega \mu_z \right) E_y, \tag{6e} \]
\[ -\omega E_z = \left( \kappa H_y + \omega (\varepsilon_{||u} - \varepsilon_{\perp u}) E_x \sin \xi \cos \xi \right) / \left[ \varepsilon_{||u} \sin^2 \xi + \varepsilon_{\perp u} \cos^2 \xi \right]. \tag{6f} \]

In these expressions, the various quantities used are given as follows:

\[ \delta_{1u} = \pm \sqrt{\left[ k_{1u}^2 - \kappa^2 \right]}, \tag{7a} \]
\[ [\varepsilon_{||u} \sin^2 \xi + \varepsilon_{\perp u} \cos^2 \xi] \delta_{2u+} \\
- \kappa(\varepsilon_{||u} - \varepsilon_{\perp u}) \sin \xi \cos \xi \\
+ \sqrt{[\varepsilon_{||u} \sin^2 \xi + \varepsilon_{\perp u} \cos^2 \xi - \kappa^2]}, \tag{7b} \]
\[ [\varepsilon_{||u} \sin^2 \xi + \varepsilon_{\perp u} \cos^2 \xi] \delta_{2u-} \\
= \kappa(\varepsilon_{||u} - \varepsilon_{\perp u}) \sin \xi \cos \xi \\
+ \sqrt{[\varepsilon_{||u} \sin^2 \xi + \varepsilon_{\perp u} \cos^2 \xi - \kappa^2]}, \tag{7c} \]
\[ J_{1u} = \delta_{1u} \omega \mu_{zu}, \tag{7d} \]
\[ J_{2u} = \omega \sqrt{[\varepsilon_{||u} \sin^2 \xi + \varepsilon_{\perp u} \cos^2 \xi - \kappa^2]}, \tag{7e} \]
\[ k_{1u}^2 = \omega^2 \mu_z E_{z1}, \tag{7f} \]
\[ k_{2u}^2 = \omega^2 \mu_z E_{z2}. \tag{7g} \]

The coefficients \( D_1 \) and \( D_2 \) represent plane waves incident on the interface, while \( C_1 \) and \( C_2 \) represent plane waves reflected off the interface.

The boundary value problem is solved by ensuring the continuity of the tangential components of the \( E \) and \( H \) fields across the interface \( z = 0 \). For a given \( \kappa \), the resulting solution is best stated in matrix notation as follows:

\[
\begin{pmatrix}
B_1 \\
B_2
\end{pmatrix} = \begin{pmatrix}
R_{11} & R_{12} \\
R_{21} & R_{22}
\end{pmatrix} \begin{pmatrix}
A_1 \\
A_2
\end{pmatrix} + \begin{pmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{pmatrix} \begin{pmatrix}
D_1 \\
D_2
\end{pmatrix}, \tag{8a}
\]

\[
\begin{pmatrix}
C_1 \\
C_2
\end{pmatrix} = \begin{pmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{pmatrix} \begin{pmatrix}
A_1 \\
A_2
\end{pmatrix} + \begin{pmatrix}
r_{11} & r_{12} \\
r_{21} & r_{22}
\end{pmatrix} \begin{pmatrix}
D_1 \\
D_2
\end{pmatrix}. \tag{8b}
\]

The various Fresnel reflection and transmission coefficients involved in the foregoing matrices are given as follows:

\[ \Delta R_{11} = -p_+ q_- + s_- \]
\[ \Delta R_{12} = 2i\eta(\delta_{2/\gamma_2}^0) p_- \]
\[ \Delta R_{11} = p_+ q_- + s_- \]
\[ \Delta R_{22} = -2i(\delta_{1/\gamma_1}) p_- \]
\[ \Delta R_{12} = 2i\eta q_{2u} q_- \]
\[ \Delta T_{11} = 4\eta(\delta_{2/\gamma_2}) u_2 \]
\[ \Delta T_{12} = -4i\eta(\delta_{2/\gamma_2}) u_1 \]
\[ \Delta T_{11} = 2\eta J_{1u} v_2 \]
\[ \Delta T_{12} = -2i\eta J_{2u} v_2 \]

where

\[ \Delta = p_+ q_- + s_+ \]
\[ q_- = (\delta_{2/\gamma_2}^0) (\delta_{1/\gamma_1}) \]
\[ s_- = 2\eta(\delta_{1/\gamma_1}) (\delta_{2/\gamma_2}) \]
\[ u_1 = \eta(\delta_{1/\gamma_1}) J_{2u} + 1 \]
\[ u_2 = \eta(\delta_{2/\gamma_2}) J_{2u} + 1 \]
\[ v_1 = \eta J_{1u} + (\delta_{1/\gamma_1}) \]
\[ v_2 = \eta J_{1u} + (\delta_{2/\gamma_2}). \]

### Analysis

Suppose now that \( D_1 = D_2 = 0 \), so that incidence is from the chiral side only. The condition on the horizontal wavenumber \( \kappa \) such that the ratio \( (B_1/B_2) \) is independent of the ratio \( (A_1/A_2) \) can be obtained easily following Chen [4], and is given by

\[ R_{12} R_{21} = R_{11} R_{22}, \tag{9} \]

which can be succinctly expressed as

\[ p_+ q_- - s_+ = 0. \tag{10} \]

Therefore, if \( \kappa \) satisfies eq. (10), the reflection ratio \( (B_1/B_2) \) is completely decorrelated from the incidence ratio \( (A_1/A_2) \).

Now, let \( A_1 = A_2 = 0 \), so that incidence is from the uniaxial side only. In order that the reflection ratio \( (C_1/C_2) \) be independent of the incidence ratio \( (D_1/D_2) \), the condition

\[ r_{12} r_{21} = r_{11} r_{22}, \tag{11} \]

must be satisfied. But, eq. (11) also boils down to \( p_+ q_- - s_+ = 0! \)

Thus, it is appropriate that eq. (10) be referred to as the Brewster reflection decorrelation condition for planar chiral-uniaxial interfaces, regardless of which half-space the incidence is from. This is a very general statement, since by setting \( \beta = 0 \), the chiral half-space can be made to be achiral; whereas by setting \( e_{||u} = e_{\perp u} \), the uniaxial half-space can be made to be isotropic. Hence, eq. (10) constitutes the chief result of this communication.

Consider also the case of normal incidence, i.e., \( \kappa = 0 \). Both \( r_{11} \) and \( r_{12} \) are directly proportional to \( q_- \). Consequently, a normally-incident ordinary (resp. extraordinary) plane wave on the uniaxial side is reflected back as an ordinary (resp. extraordinary) plane wave. On the other hand, \( R_{11} \) and \( R_{12} \) are not zero when \( \kappa = 0 \). Hence, even for a normally incident left- (resp. right-) circularly polarized plane wave, the reflected field will have both left- and right-circularly polarized components (unless \( \xi = 0 \)); this is a direct consequence of the anisotropy of the uniaxial medium.

Finally, the following two relationships should also be noted:
\[
\begin{align*}
[(R_{12}/i\eta) + (i\eta R_{21})] & \left[1 - R_{11} R_{22} + (R_{12}/i\eta) (i\eta R_{21})\right]^{-1} \\
& = \rho_- / \rho_+ , \\
[\rho_{11} - \rho_{22}] & \left[1 - \rho_{11} \rho_{22} + \rho_{12} \rho_{21}\right]^{-1} = -s_- / s_+ .
\end{align*}
\] (12a, 12b)

These two relations are in the same vein as the relations between Fresnel reflection coefficients derived by Azzam [6] for planar dielectric-dielectric interfaces. Since the uniaxial medium is anisotropic, the left sides of both eqs. (12a) and (12b) contain \(\kappa\); corresponding to the case investigated by Azzam, the left sides would not contain \(\kappa\) and be independent of the angle of incidence.

References

[5] The optic axis of the uniaxial medium should be represented by \(\epsilon = \epsilon_0 \sin \xi \cos \zeta + \epsilon_1 \sin \zeta \sin \zeta + \epsilon_2 \cos \xi\) \(0^\circ \leq \zeta \leq 180^\circ, 0^\circ \leq \xi \leq 360^\circ\), in all generality. However, a simple rotation of the co-ordinate system about the \(\zeta\)-axis reduces \(\epsilon\) to the form given as eq. (5).