PHYS 4100: Intro to Quantum Mechanics — Homework 11 Full name: ________________
April 22$^{\text{th}}$, 2021 This HW is worth a total of **20 points**.

This homework relates to time-independent perturbation theory. Here, we will examine the cases of non-degenerate and degenerate perturbative approach for the two-dimensional harmonic oscillator.

Our starting point is the isotropic harmonic oscillator in *two* dimensions. As shown in Chapter 10, this *unperturbed* Hamiltonian is given by

$$H_0 = \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_y^2}{2m} + \frac{m\omega^2}{2}(\hat{x}^2 + \hat{y}^2)$$

1. What are the energies of the three lowest-lying states? Clearly explain the degeneracy. For this question, adopt the technique used in chapter 10 where we used Cartesian coordinates to show that the x and y variables can be effectively separated, showing that the 2D oscillator is simply the sum of two isolated 1D harmonic oscillators. [2]

2. We will now consider the following perturbation, which consists in breaking the isotropy of the Hamiltonian

$$\hat{H}_1 = \Delta k \hat{x}^2$$

   (a) The new Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_1$ can be solved exactly using same technique as described above. What is its energy spectrum? [2]

   (b) Use perturbation theory to calculate the effect of $\hat{H}_1$ on the ground state at first order for energy and eigenstate. [2]

   (c) Use perturbation theory to calculate the effect of $\hat{H}_1$ on the ground state at second order for energy. [2]

   (d) Calculate effect of perturbation on first-excited state (degenerate!) of $\hat{H}_0$. [4]

3. Suppose now that we apply a different perturbation to the isotropic harmonic oscillator

$$\hat{H}_1 = m\alpha^2 \hat{x}\hat{y}$$

   (a) Find the zeroth-order energy eigenket and the corresponding energy to first order for each of the three lowest-lying states. [6]

   (b) Draw an energy diagram with and without the perturbation for the three energy states. Make sure to specify which unperturbed state is connected to which perturbed state. [2]