PHYS 4100: Intro to Quantum Mechanics — Quiz 5 **Answer Key**
Dr. Vincent Meunier, February 25, 2021 This quiz is worth a total of **10 points**.

Answer the following questions succinctly.

1. Provide an expression of the operator of time translation for infinitesimal time dt. [2]

Answer:

$$\hat{U}(dt) = 1 - i\hat{H}dt/\hbar$$

2. Show that the generator of time translation is Hermitian. [2]

Answer: Since $\hat{U}(dt)$ is unitary, the hermiticity of $\hat{H}$ follows from

$$(1 - i\hat{H}dt/\hbar)(1 + i\hat{H}^\dagger dt/\hbar) = 1,$$

after dropping negligible terms in $O(dt^2)$.

3. Provide an expression of the time translation operator in the case when the generator of time translation is time *independent*. [2]

Answer:

$$\hat{U}(t) = e^{-i\hat{H}t/\hbar}$$

4. Suppose $|E\rangle$ is an eigenstate of the generator of time translation, with eigenvalue E. How does this state evolve with time? (you can use result from question 1 or question 3). [4]

Answer:

$$\begin{aligned}\hat{U}(t)\,|E\rangle &= e^{-i\hat{H}t/\hbar}\,|E\rangle \\ &= e^{-i\hat{E}t/\hbar}\,|E\rangle\end{aligned}$$

Therefore we see that the state only changes by an overall phase factor. In other words, the state remains the same, it is *stationary*.

5. For a time independent generator of time translation, the expectation value of the energy of a system is conserved [2 B]
A. True B. False
Use the expression of the derivative of the expectation value to justify your result. (no credit if answer is not fully justified)

Answer: Since, for any (not-explicitly time-dependent) operator $\hat{A}$, we have

$$\frac{d}{dt}\langle\Psi|\hat{A}|\Psi\rangle = \frac{i}{\hbar}\langle\Psi|\left[\hat{A},\hat{H}\right]|\Psi\rangle\,,$$

it is clear that if $\hat{A} = \hat{H}$, then $\frac{d}{dt}\langle\Psi|\hat{H}|\Psi\rangle = 0$ for any state vector $|\Psi\rangle$ since all operators commute with themselves.