Answer the following questions succinctly.

1. Provide an expression of the operator of time translation for infinitesimal time dt.

Answer:

$$\hat{U}(dt) = 1 - i\hat{H}dt/\hbar$$

2. Show that the generator of time translation is Hermitian.

Answer: Since $\hat{U}(dt)$ is unitary, the hermiticity of \hat{H} follows from

$$(1 - i\hat{H}dt/\hbar)(1 + i\hat{H}^{\dagger}dt/\hbar) = 1,$$

after dropping negligible terms in $O(dt^2)$.

3. Provide an expression of the time translation operator in the case when the generator of time translation is time *independent*.

Answer:

$$\hat{U}(t) = e^{-i\hat{H}t/\hbar}$$

4. Suppose $|E\rangle$ is an eigenstate of the generator of time translation, with eigenvalue E. How does this state evolve with time? (you can use result from question 1 or question 3).

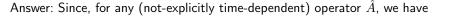
Answer:

$$\hat{U}(t) |E\rangle = e^{-iHt/\hbar} |E\rangle$$
$$= e^{-i\hat{E}t/\hbar} |E\rangle$$

Therefore we see that the state only changes by an overall phase factor. In other words, the state remains the same, it is *stationary*.

5. For a time independent generator of time translation, the expectation value of the energy of a system is conserved 2 B A. True B. False

Use the expression of the derivative of the expectation value to justify your result. (no credit if answer is not fully justified)



$$\frac{d}{dt} \left< \Psi | \hat{A} | \Psi \right> = \frac{i}{\hbar} \left< \Psi | \left[\hat{A}, \hat{H} \right] | \Psi \right>,$$

it is clear that if $\hat{A} = \hat{H}$, then $\frac{d}{dt} \langle \Psi | \hat{H} | \Psi \rangle = 0$ for any state vector $|\Psi \rangle$ since all operators commute with themselves.

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