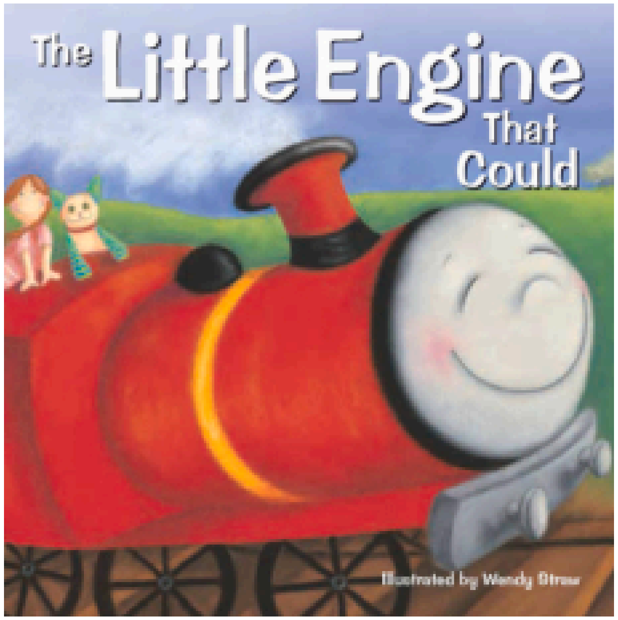
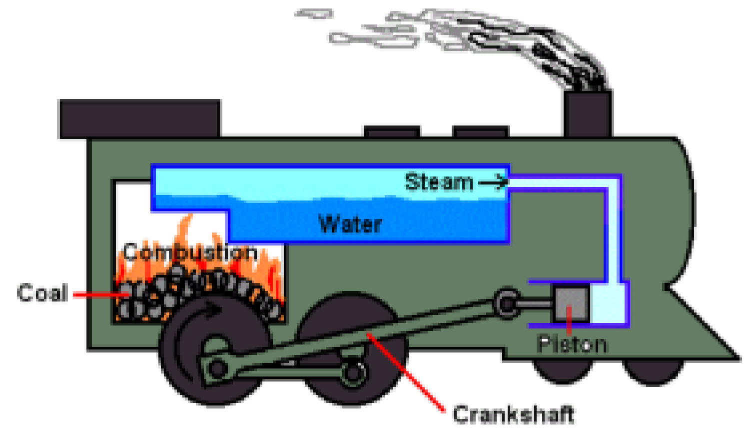
Lecture 13. *Heat Engines and the second law*

- In this unit we will learn about
  - Heat engines
  - Different statements of the second law
  - Carnot cycle
  - Most efficient engine



Heat Engine

- **"Heat engines"** are machines that produce work from a temperature difference between two reservoirs
- **Reservoir:** large body that keeps its temperature regardless of amount of heat taken away from it (infinite heat capacity)*



External Combustion Engine

*to be contrasted with a thermometer

https://www.pinterest.co.uk/pin/836543699514990870/

The second law of thermodynamics

- About the direction of heat flow that occurs as a system approaches equilibrium (“arrow of time”).

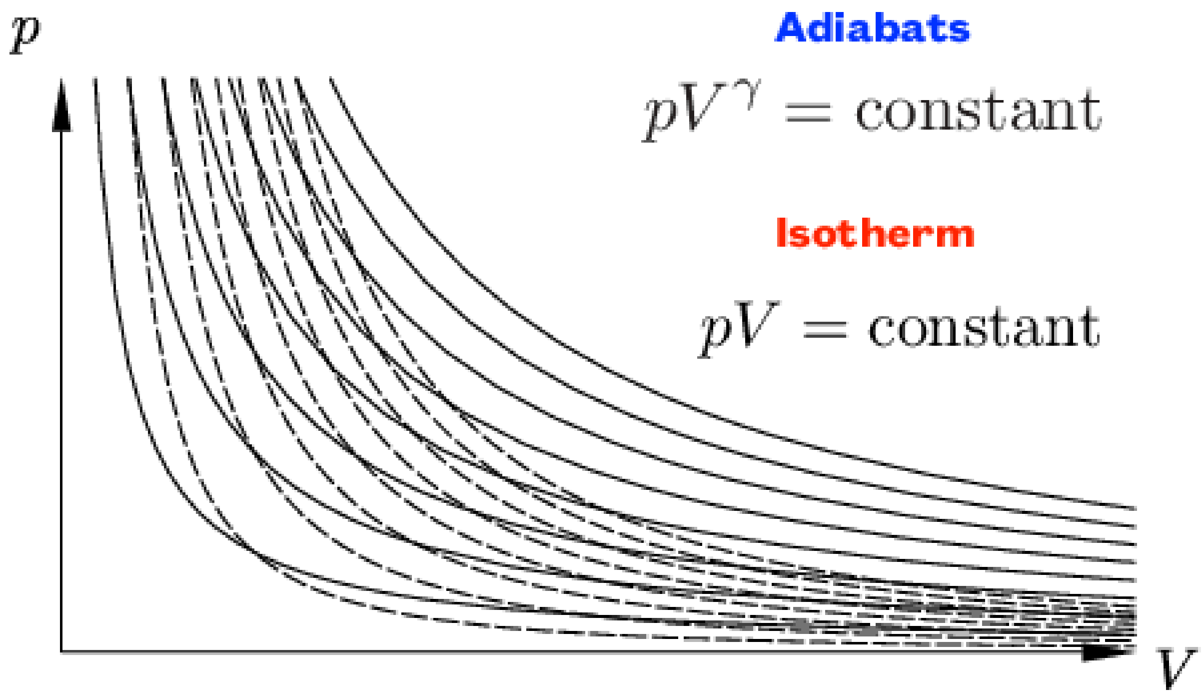
Clausius' statement:

No process is possible whose sole result is the transfer of heat from a colder to a hotter body.

$\Leftarrow$

The Carnot engine

- How good can conversion of heat to work be?
- **Engine:** system operating a cyclic process that converts heat into work. It has to be cyclic so that it can be continuously operated, producing a steady power.



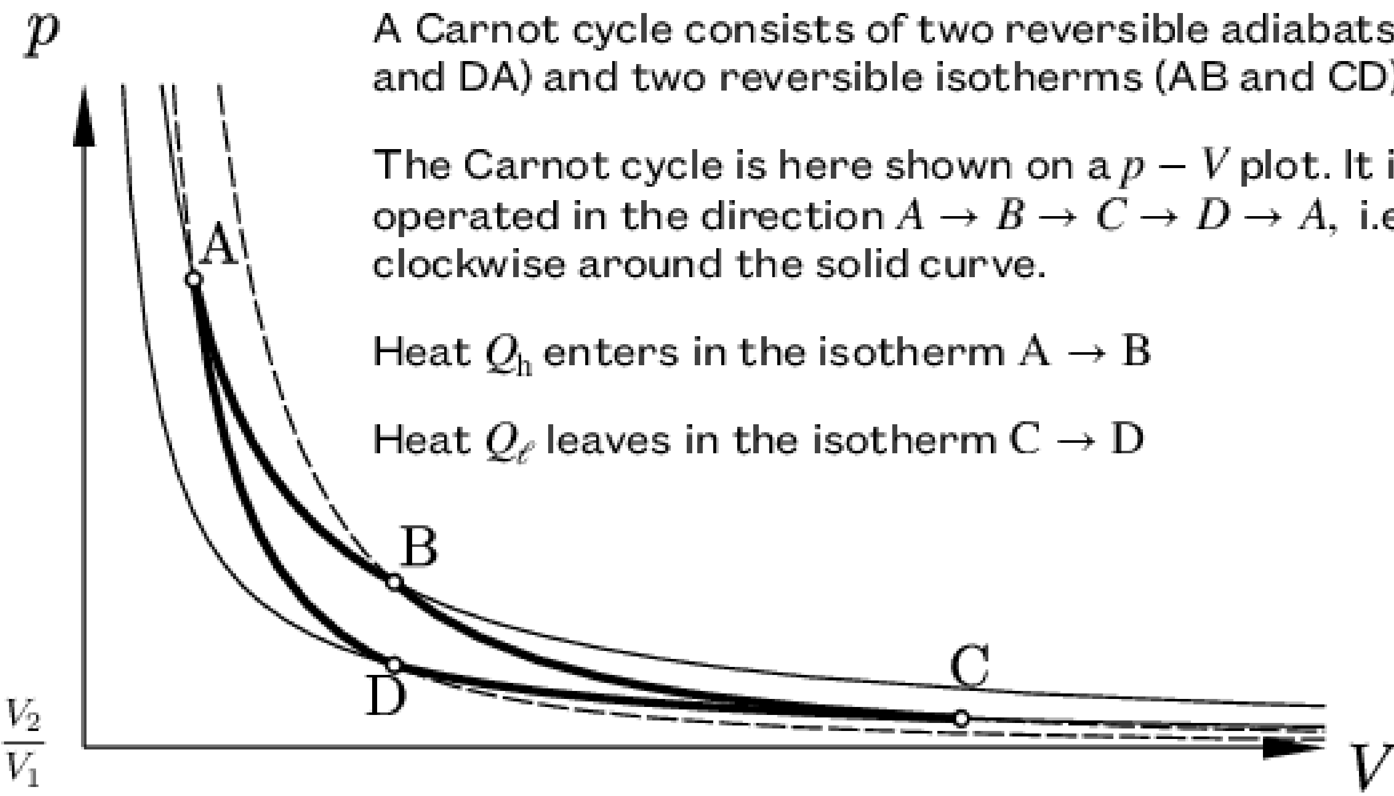
Adiabats

$$pV^{\gamma} = \text{constant}$$

Isotherm

$$pV = \text{constant}$$

Carnot cycle



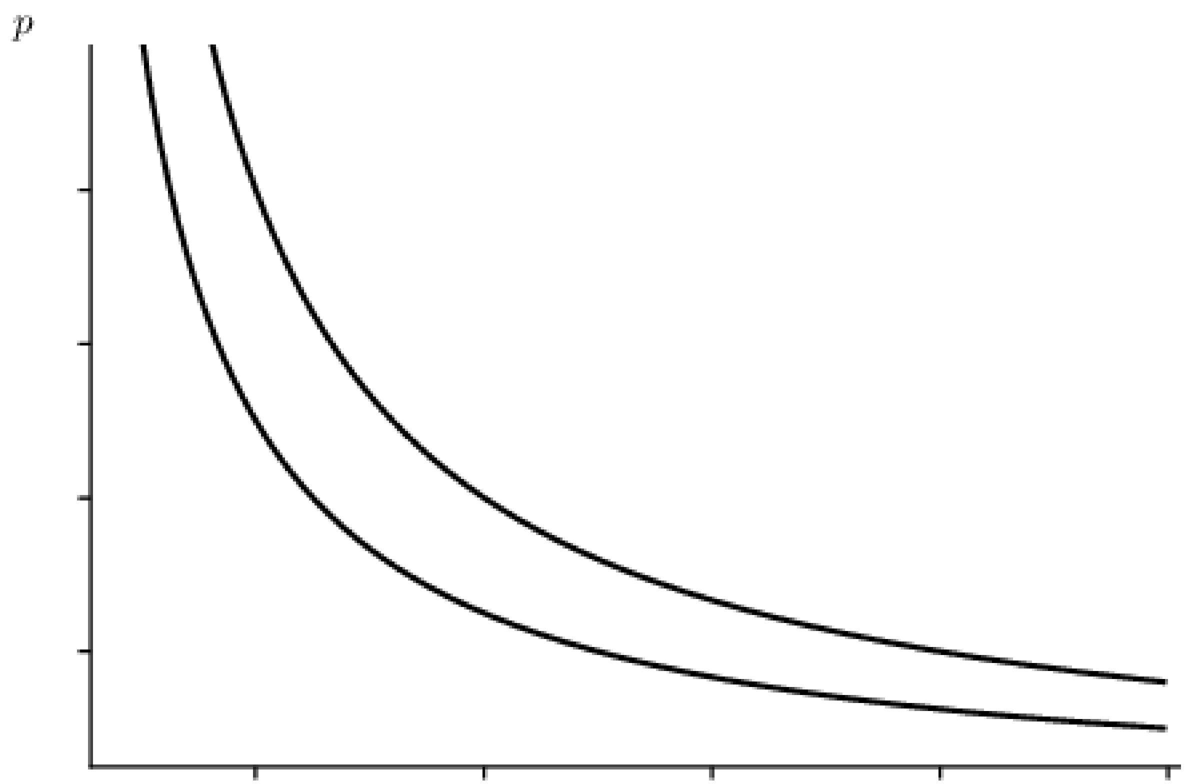
$$\Delta Q = RT \ln \frac{V_2}{V_1}$$

A Carnot cycle consists of two reversible adiabats (BC and DA) and two reversible isotherms (AB and CD).

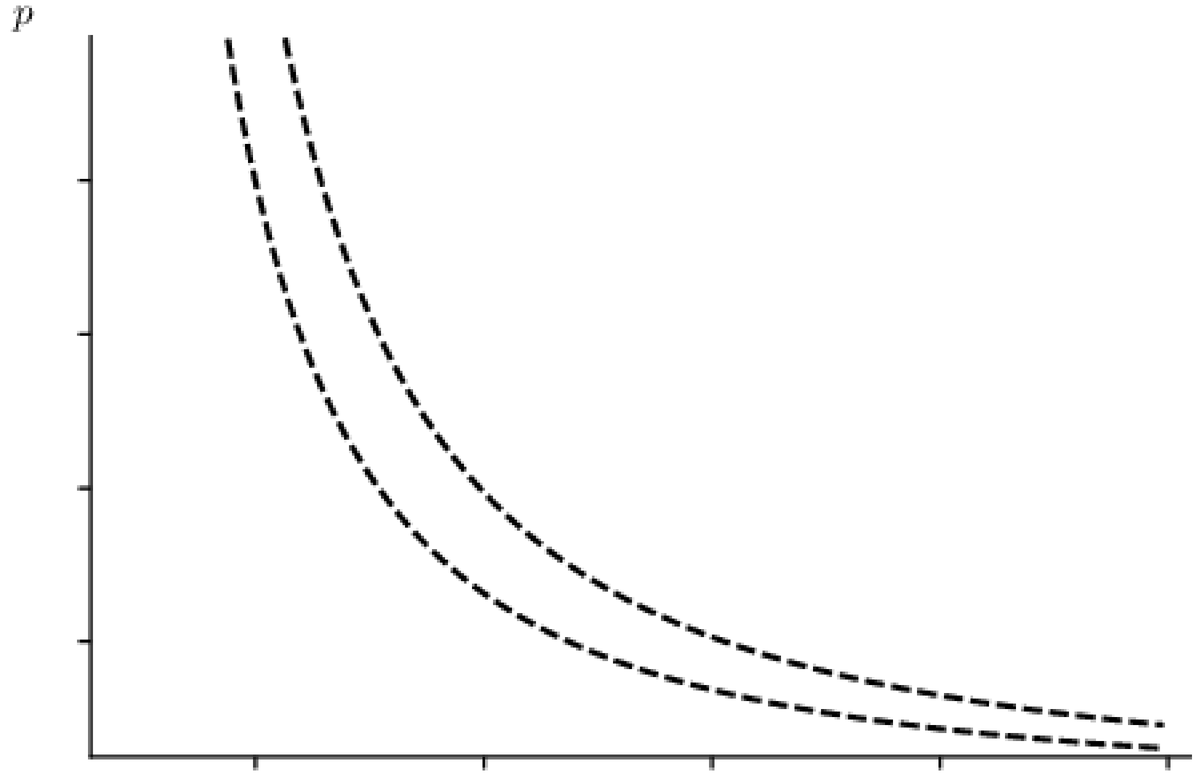
The Carnot cycle is here shown on a $p - V$ plot. It is operated in the direction $A \to B \to C \to D \to A,$ i.e., clockwise around the solid curve.

Heat Q_{h} enters in the isotherm $\mathrm{A} \to \mathrm{B}$

Heat Q_{ℓ} leaves in the isotherm $\mathrm{C} \to \mathrm{D}$



p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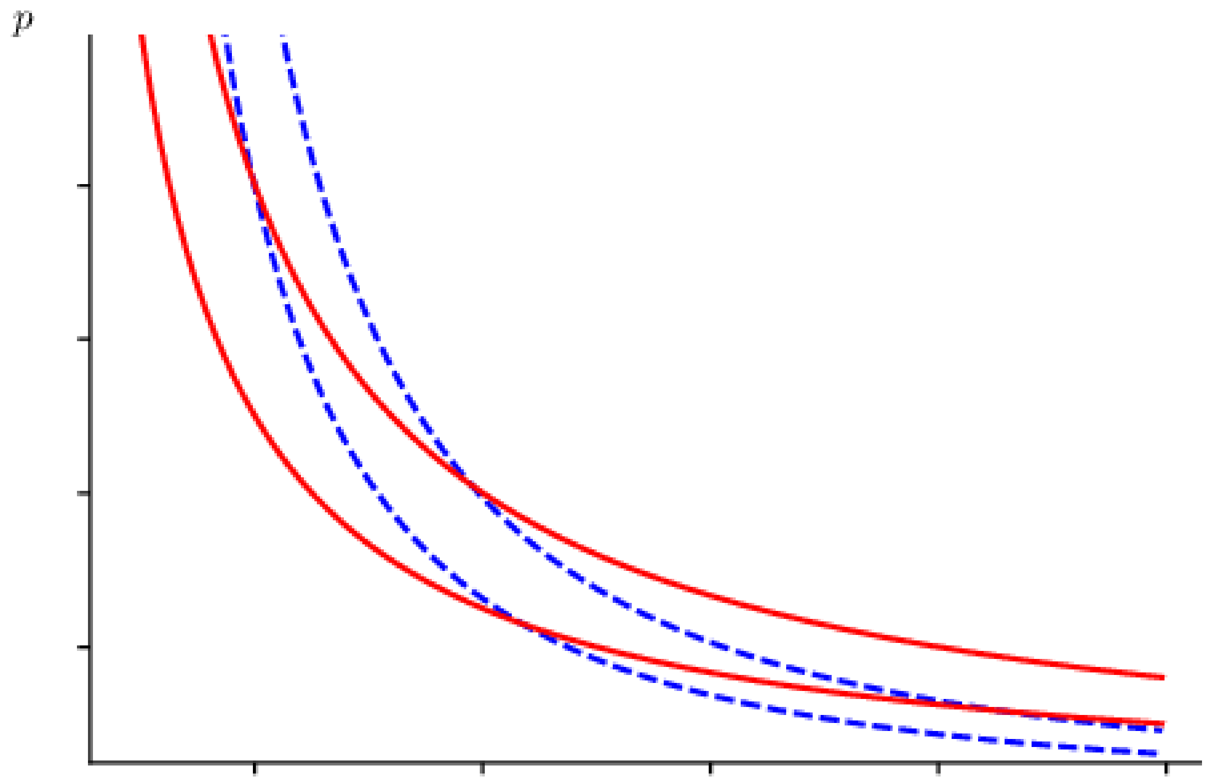
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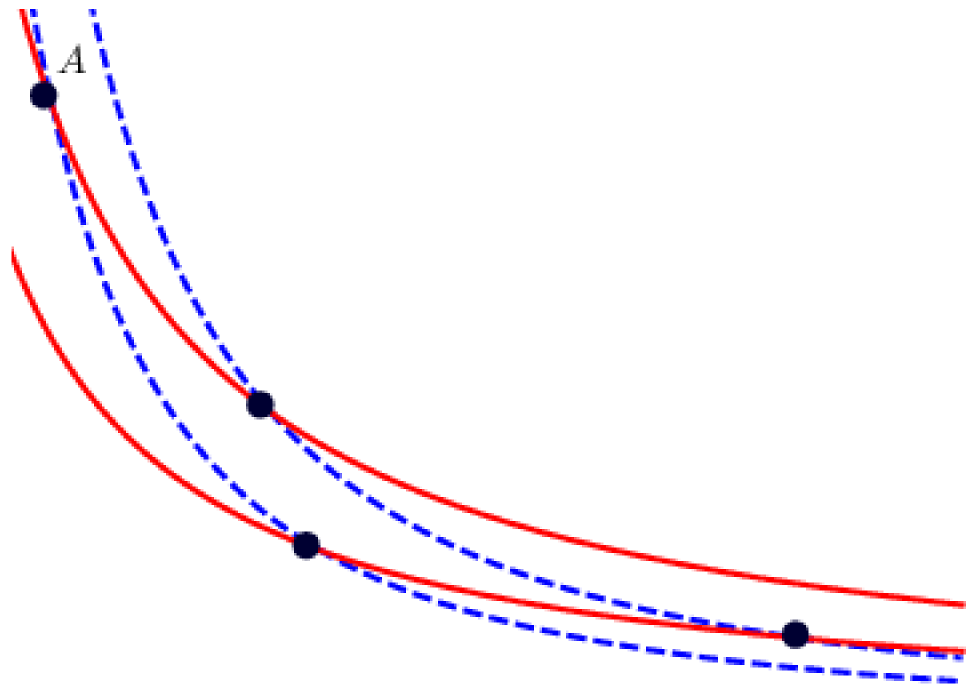
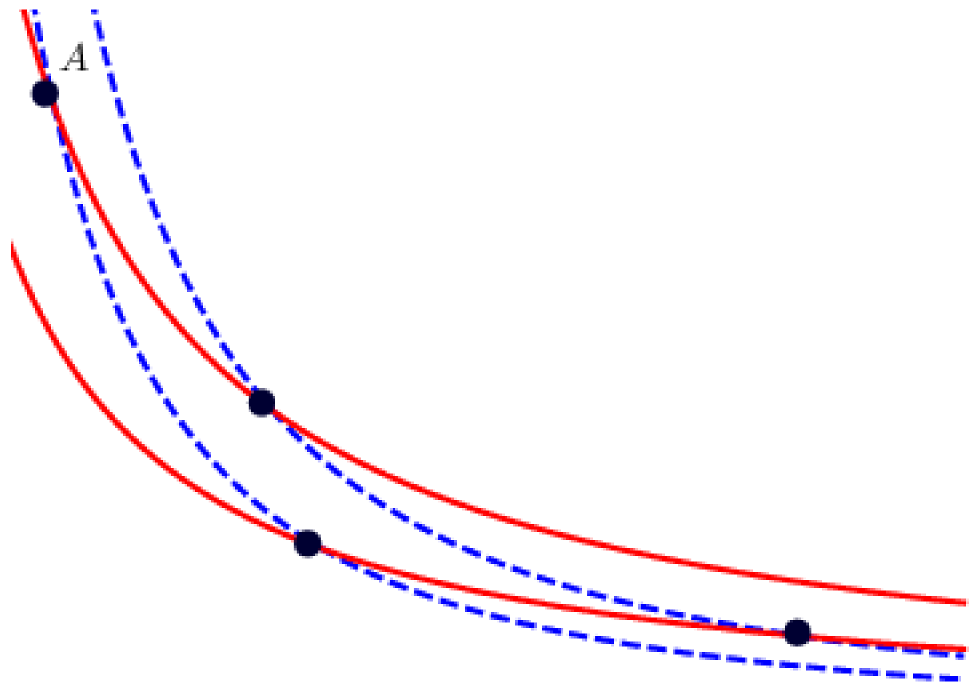
(reversible) Adiabats

(reversible) Isotherm



Ideal gas

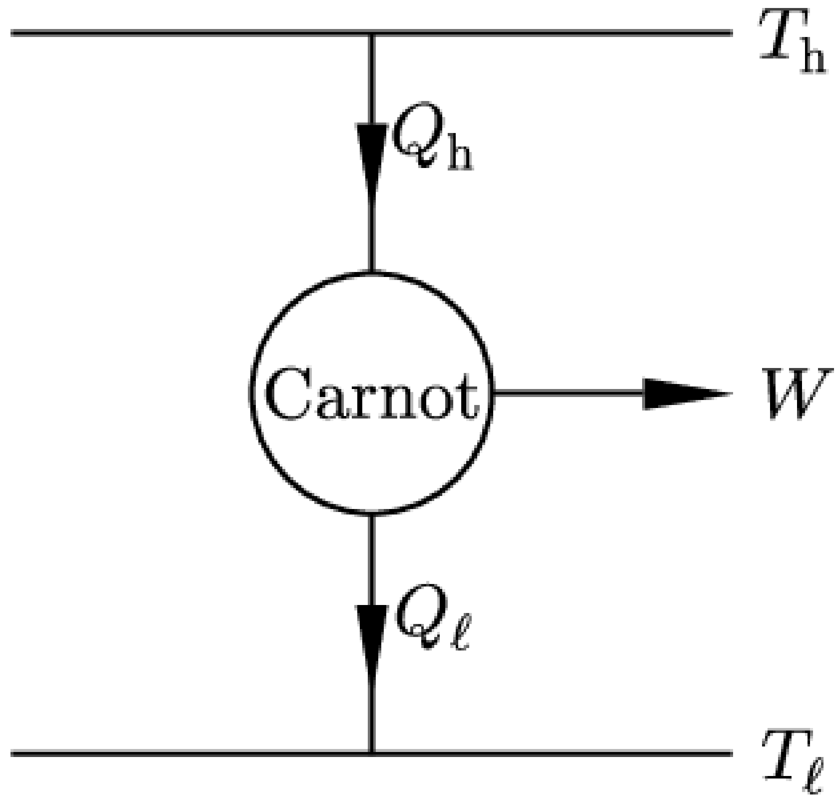
$$
\begin{aligned}
\mathrm{A} &\rightarrow \mathrm{B}: & Q_{\mathrm{h}} &= RT_{\mathrm{h}} \ln \frac{V_{\mathrm{B}}}{V_{\mathrm{A}}}, \\
\mathrm{B} &\rightarrow \mathrm{C}: & \left(\frac{T_{\mathrm{h}}}{T_{\ell}}\right) &= \left(\frac{V_{\mathrm{C}}}{V_{\mathrm{B}}}\right)^{\gamma-1}, \\
\mathrm{C} &\rightarrow \mathrm{D}: & Q_{\ell} &= -RT_{\ell} \ln \frac{V_{\mathrm{D}}}{V_{\mathrm{C}}}, \\
\mathrm{D} &\rightarrow \mathrm{A}: & \left(\frac{T_{\ell}}{T_{\mathrm{h}}}\right) &= \left(\frac{V_{\mathrm{A}}}{V_{\mathrm{D}}}\right)^{\gamma-1}.
\end{aligned}
$$



Efficiency

Efficiency is the ratio of "what you want to achieve' to "what you have to do to achieve it".

$$\eta = \frac{W}{Q_{\mathrm{h}}}$$



Efficiency of the Carnot Engine

$$\eta_{\text{Carnot}} = \frac{Q_{\text{h}} - Q_{\ell}}{Q_{\text{h}}}$$

Example:

A power station steam turbine operates between $T_{\text{h}} \sim 800\text{K}$ and $T_{\ell} = 300\text{K}$.

If it were a Carnot engine, it could achieve an efficiency of $\eta_{\text{Carnot}} = (T_{\text{h}} - T_{\ell})/T_{\text{h}} \approx 60\,\%$

Real power stations do not achieve the maximum efficiency but rather around $40\,\%$

Real engines are less efficient than Carnot engines!

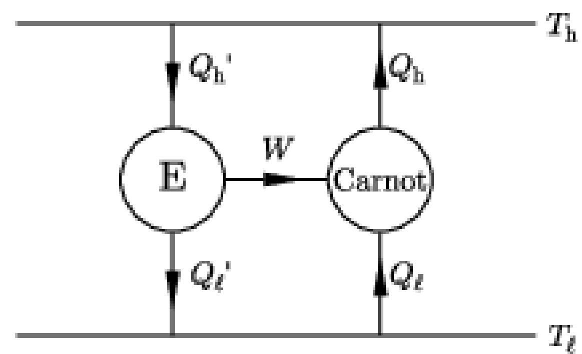
Carnot theorem

Of all the heat engines working between two given temperatures, none is more efficient than a Carnot engine.

Proof

Imagine that E with $\eta_{\rm E} > \eta_{\rm Carnot}$. The Carnot engine is reversible so one can run it in reverse. We connect engine E and a Carnot engine (run in reverse)

$\eta_{\rm E} > \eta_{\rm Carnot}$



The first law of thermodynamics implies that

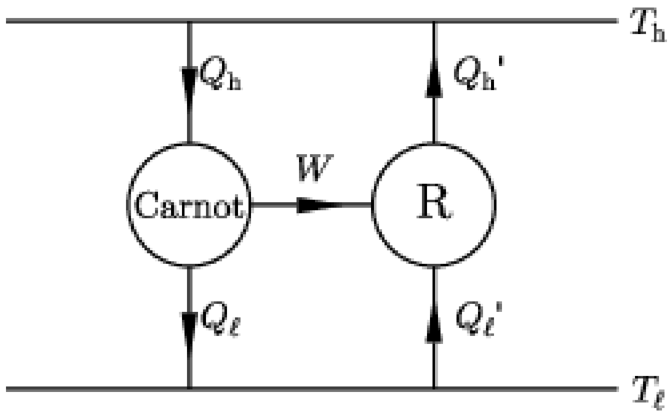
$W = Q'_{\rm h} - Q'_\ell = Q_{\rm h} - Q_\ell$ so that

- is positive and so is !
- is the net amount of heat dumped into the reservoir at temperature
- is the net amount of heat extracted from the reservoir at temperature

The combined system extracts heat from the reservoir at and dumps it into the reservoir at

Corollary: All reversible engines working between two temperatures have the same efficiency $\eta_{\rm Carnot}$

- Take another reversible engine R.
  - Its efficiency $\eta_{\rm R} \leq \eta_{\rm Carnot}$ (Carnot's theorem)
  - We run it in reverse and connect it to a Carnot engine going forwards.
- This arrangement will simply transfer heat from the cold reservoir to the hot reservoir and violates Clausius' statement of the second law of thermodynamics unless $\eta_{\rm R} = \eta_{\rm Carnot}$.
- Therefore all reversible engines have the same efficiency



Equivalence of Clausius an Kelvin's statements (1)

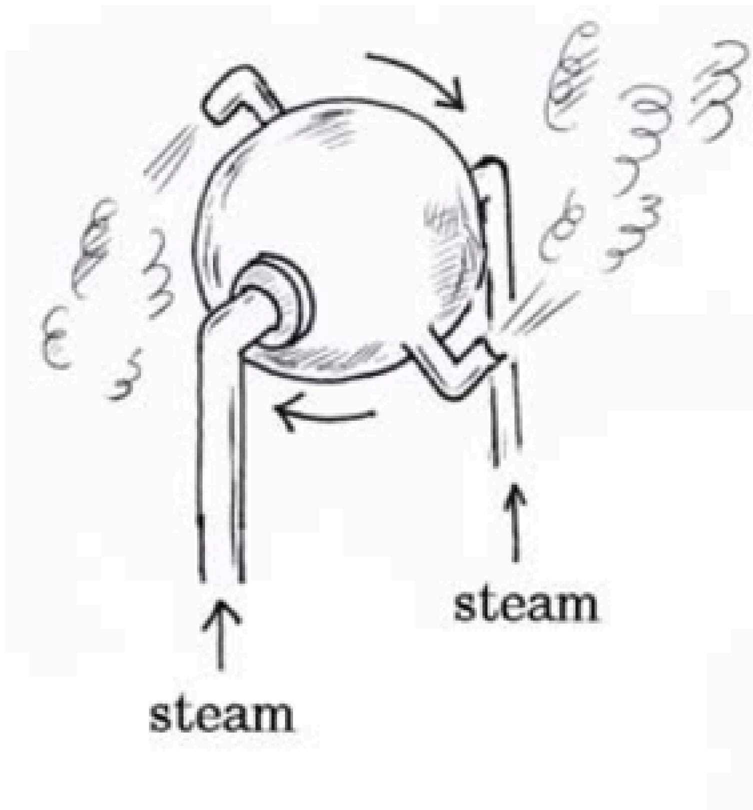
$$\Rightarrow$$

Equivalence of Clausius an Kelvin's statements (2)

$$\Rightarrow$$

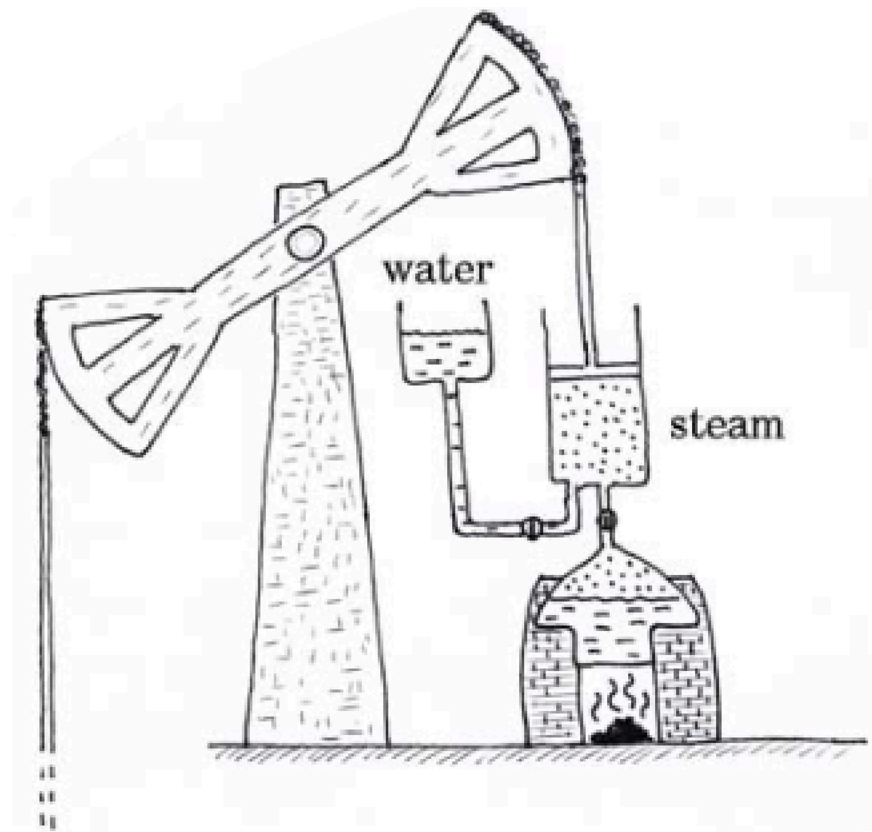
Heat engines

- Hero of Alexandria (almost 2,000 years ago)
- Heat engine: convert heat into work



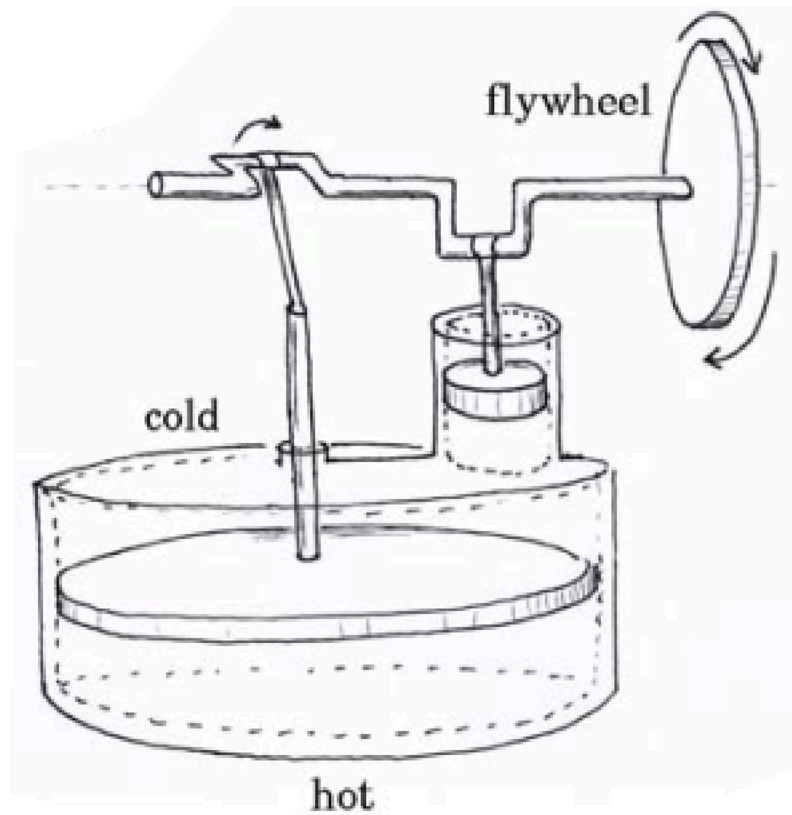
Heat engines

- Thomas Newcomen (1664-1729)
- Heat engine: convert heat into work
- Pumping water from mines



Heat engines

- James Watt (1736 — 1819)
- Robert Stirling (1790 — 1878)
  - Heat engine: convert heat into work
  - Improved concept: condensation takes place in a separate chamber

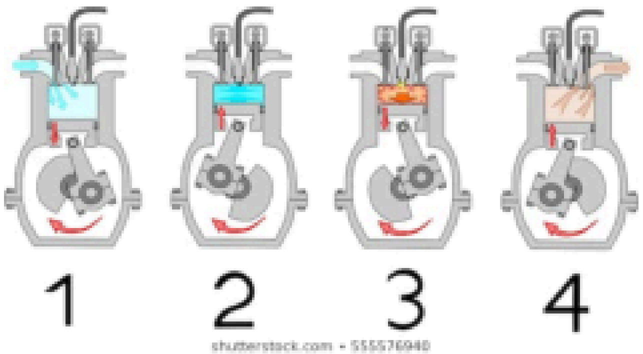


Internal combustion engine

Rather than externally heating water to produce steam (as with Newcomen's and Watt's engines) or to produce a temperature differential (as with Stirling's engine), here the burning of fuel inside the engine's combustion chamber generates the high temperature and pressure necessary to produce useful work.

There are many different types of internal combustion engines, including

- **piston engines** (in which pressure is converted into rotating motion using a set of pistons),
- **combustion turbines** (in which gas flow is used to spin a turbine's blades) and
- **jet engines** (in which a fast moving jet of gas is used to generate thrust).

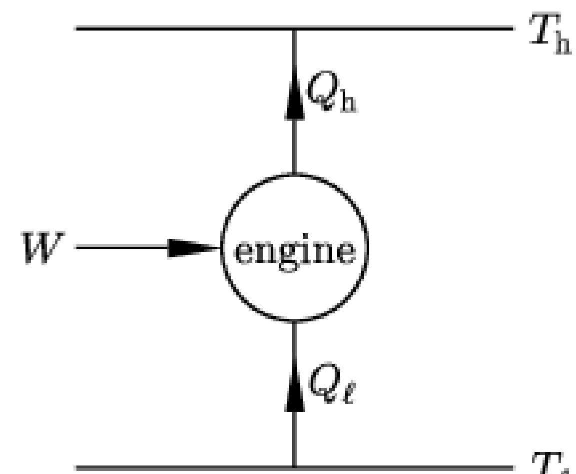


Heat Engine running backwards

- The **refrigerator** is a heat engine that is run backwards so that you put work in and cause a heat flow from a cold reservoir to a hot reservoir.
  - cold reservoir is the food
  - hot reservoir is the room.
- For a refrigerator, we must define the efficiency in a different way from the efficiency of a heat engine. This is because what you want to achieve is "heat sucked out of the contents of the refrigerator"
- We define the efficiency of a refrigerator as

$$\eta = \frac{Q_\ell}{W}$$

Can be above 100%

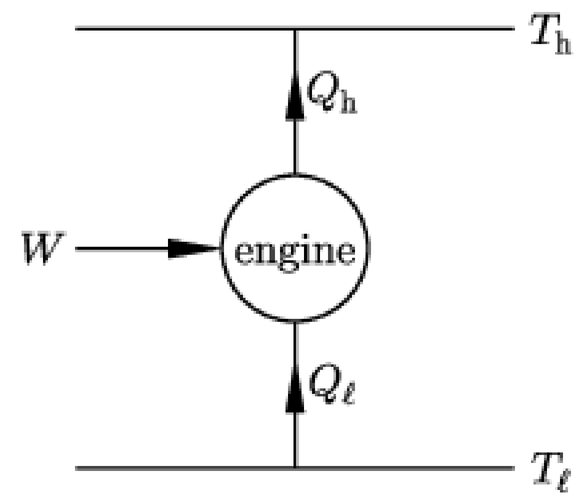


Heat Engine running backwards

- A **heat pump** is essentially a refrigerator but it is utilized in a different way. It is used to pump heat from a reservoir, to a place where it is desired to add heat.
- The efficiency of a heat pump is defined as

$$\eta = \frac{Q_{\mathrm{h}}}{W}$$

Always above 100%



This shows why heat pumps are attractive for heating. It is always possible to turn work into heat with $100\,\%$ efficiency (an electric fire turns electrical work into heat in this way), but a heat pump can allow you to get even more heat into your house for the same electrical work

Clausius' theorem

Carnot engine

- Heat is not a conserved quantity. Can we tell more about the process?

$$\frac{Q_{\rm h}}{Q_\ell} = \frac{T_{\rm h}}{T_\ell}. \qquad \textbf{Carnot engine}$$

- However, if we look at the heat entering the system (>0) , we see that, for the cycle:
- If we consider infinitesimally small steps:

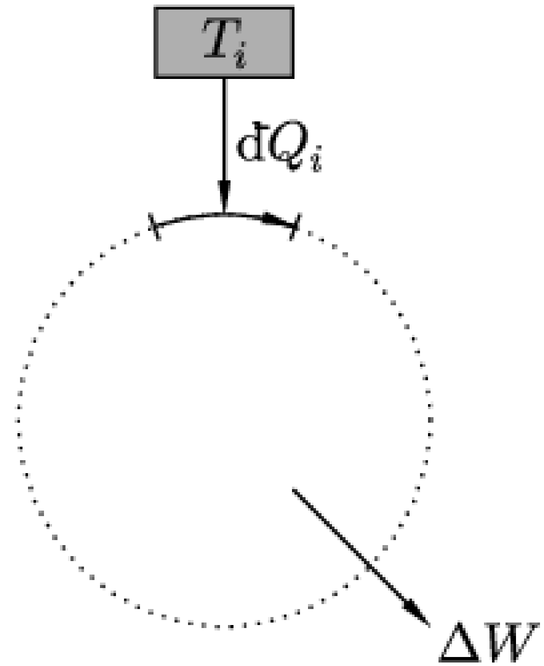
$$\sum_{\rm cycle} \frac{\Delta Q_{\rm rev}}{T} = \frac{Q_{\rm h}}{T_{\rm h}} + \frac{(-Q_\ell)}{T_\ell} = 0$$

$$\Longrightarrow \quad \oint \frac{đQ_{\rm rev}}{T} = 0$$

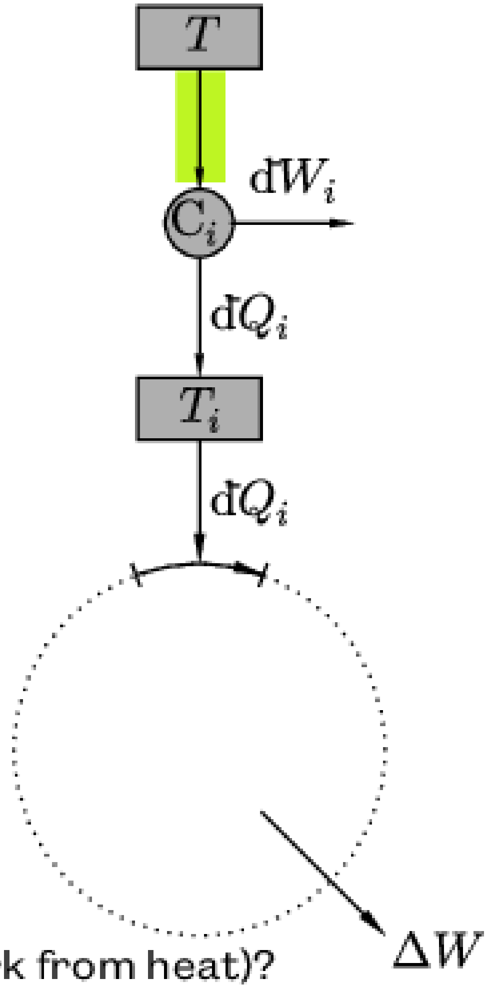
General cycle

For this cycle, heat $đQ_i$ enters at a particular part of the cycle. At this point the system is connected to a reservoir, which is at temperature The total work extracted from the cycle is given by

from the first law of thermodynamics. The sum here is taken around the whole cycle, indicated schematically by the dotted circle.



Next we imagine that the heat at each point is supplied via a Carnot engine, which is connected between a reservoir at temperature T and the reservoir at temperature T_i. The reservoir at T is common for all the Carnot engines connected at all points of the cycle. Each Carnot engine produces work $đW_i$, and for a Carnot engine we know that



$$\frac{\text{heat to reservoir at } T_i}{T_i} = \frac{\text{heat from reservoir at } T}{T}$$

$$\Longrightarrow \frac{đQ_i}{T_i} = \frac{đQ_i + đW_i}{T}$$

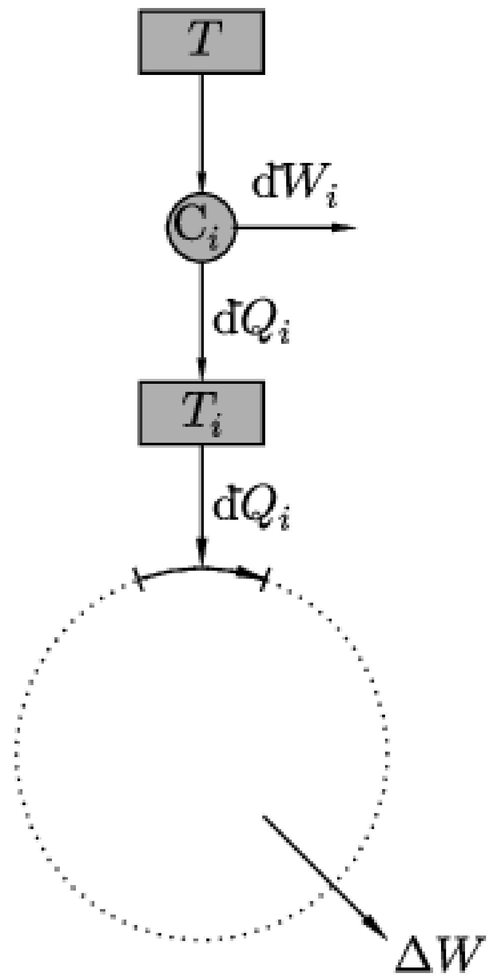
Violates second law of thermo? (is the sole effect to produce work from heat)?

total work produced per cycle $= \Delta W + \sum_{\text{cycle}} đW_i \leq 0.$

$$đW_i = đQ_i \left(\frac{T}{T_i} - 1 \right)$$

$$\Delta W = \sum_{\text{cycle}} đQ_i$$

$$\Longrightarrow \qquad T \sum_{\text{cycle}} \frac{đQ_i}{T_i} \leq 0.$$



Clausius inequality

(equality for reversible cycle)

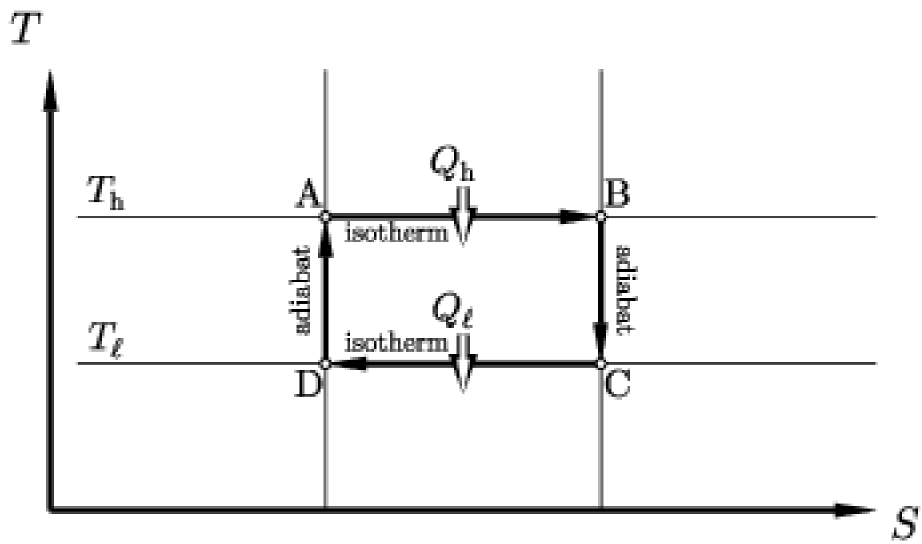
Clausius theorem

For any closed cycle,

$$\oint \frac{đQ}{T} \leq 0$$

where equality necessarily holds for a reversible cycle

Back to Carnot Cycle



$$\frac{Q_h}{Q_\ell} = \frac{T_h}{T_\ell}$$

Summary

- **Equivalent statements of second law:**
  - *No process is possible whose sole result is the transfer of heat from a colder to a hotter body.* (**Clausius' statement of the second law of thermodynamics**)
  - *No process is possible whose sole result is the complete conversion of heat into work.* (**Kelvin's statement of the second law of thermodynamics**)
  - *Of all the heat engines working between two given temperatures, none is more efficient than a Carnot engine.* (**Carnot's theorem**)
- All reversible engines operating between temperatures T_h and T_ℓ have the efficiency of a Carnot engine: $\eta_{\text{Carnot}} = (T_\mathrm{h} - T_\ell)/T_\mathrm{h}$
  - For a Carnot engine: $\dfrac{Q_\mathrm{h}}{Q_\ell} = \dfrac{T_\mathrm{h}}{T_\ell}$
- **Clausius' theorem** states that for any closed cycle, where equality necessarily holds for a reversible cycle.